INTERNATIONAL STANDARD

ISO 281

Second edition 2007-02-15

Rolling bearings — Dynamic load ratings and rating life

Roulements — Charges dynamiques de base et durée nominale



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Foreword

ISO (the International Organization for Standardization) is a worldwide federation of national standards bodies (ISO member bodies). The work of preparing International Standards is normally carried out through ISO technical committees. Each member body interested in a subject for which a technical committee has been established has the right to be represented on that committee. International organizations, governmental and non-governmental, in liaison with ISO, also take part in the work. ISO collaborates closely with the International Electrotechnical Commission (IEC) on all matters of electrotechnical standardization.

International Standards are drafted in accordance with the rules given in the ISO/IEC Directives, Part 2.

The main task of technical committees is to prepare International Standards. Draft International Standards adopted by the technical committees are circulated to the member bodies for voting. Publication as an International Standard requires approval by at least 75 % of the member bodies casting a vote.

Attention is drawn to the possibility that some of the elements of this document may be the subject of patent rights. ISO shall not be held responsible for identifying any or all such patent rights.

ISO 281 was prepared by Technical Committee ISO/TC 4, Rolling bearings, Subcommittee SC 8, Load ratings and life.

This second edition cancels and replaces the first edition (ISO 281:1990), ISO 281:1990/Amd. 1:2000, ISO 281:1990/Amd. 2:2000 and ISO/TS 16799:1999, which have been technically revised.

Introduction

It is often impractical to establish the suitability of a bearing selected for a specific application by testing a sufficient number of bearings in that application. However, life, as defined in 3.1, is a primary representation of the suitability. A reliable life calculation is therefore considered to be an appropriate and convenient substitute for testing. The purpose of this International Standard is to provide the required basis for life calculation.

Since ISO 281 was published in 1990, additional knowledge has been gained regarding the influence on bearing life of contamination, lubrication, internal stresses from mounting, stresses from hardening, fatigue load limit of the material, etc. In ISO 281:1990/Amd. 2:2000, a general method was presented to consider such influences in the calculation of a modified rating life of a bearing. This amendment is incorporated in this International Standard, which also provides a practical method to consider the influence on bearing life of lubrication condition, contaminated lubricant and fatigue load of bearing material.

ISO/TS 16281 ^[1] introduced advanced calculation methods which enable the user to take into account the influence on bearing life of bearing-operating clearance and misalignment under general loading conditions. The user can also consult the bearing manufacturer for recommendations and evaluation of equivalent load and life for these operation conditions and other influences as, for example, rolling element centrifugal forces or other high-speed effects.

Calculations according to this International Standard do not yield satisfactory results for bearings subjected to such application conditions and/or of such internal design which result in considerable truncation of the area of contact between the rolling elements and the ring raceways. Unmodified calculation results are thus not applicable, for example, to ball bearings with filling slots that project substantially into the ball/raceway contact area when the bearing is subjected to axial loading in the application. Bearing manufacturers should be consulted in such cases.

The life modification factors for reliability, a₁, have been slightly changed and extended to 99,95 % reliability.

Revisions of this document will be required from time to time, as the result of new developments or in the light of new information concerning specific bearing types and materials.

Background information regarding the derivation of equations and factors in this document is given in $ISO/TR~8646^{1)}$ and $ISO/TR~1281-2^{[2]}$.

¹⁾ Under revision. Will be published under the reference ISO/TR 1281-1.

Rolling bearings — Dynamic load ratings and rating life

1 Scope

This International Standard specifies methods of calculating the basic dynamic load rating of rolling bearings within the size ranges shown in the relevant ISO publications, manufactured from contemporary, commonly used, high quality hardened bearing steel, in accordance with good manufacturing practice and basically of conventional design as regards the shape of rolling contact surfaces.

This document also specifies methods of calculating the basic rating life, which is the life associated with 90 % reliability, with commonly used high quality material, good manufacturing quality and with conventional operating conditions. In addition, it specifies methods of calculating the modified rating life, in which various reliabilities, lubrication condition, contaminated lubricant and fatigue load of the bearing are taken into account.

This International Standard does not cover the influence of wear, corrosion and electrical erosion on bearing life.

This document is not applicable to designs where the rolling elements operate directly on a shaft or housing surface, unless that surface is equivalent in all respects to the bearing ring (or washer) raceway it replaces.

Double-row radial bearings and double-direction thrust bearings are, when referred to in this document, presumed to be symmetrical.

Further limitations concerning particular types of bearings are included in the relevant clauses.

2 Normative references

The following referenced documents are indispensable for the application of this document. For dated references, only the edition cited applies. For undated references, the latest edition of the referenced document (including any amendments) applies.

ISO 76, Rolling bearings — Static load ratings

ISO 5593, Rolling bearings — Vocabulary

ISO/TR 8646:1985, Explanatory notes on ISO 281/1-1977²)

ISO 15241, Rolling bearings — Symbols for quantities

²⁾ Under revision. Will be published under the reference ISO/TR 1281-1.

3 Terms and definitions

For the purposes of this document, the terms and definitions given in ISO 5593 and the following apply.

3.1

life

(of an individual rolling bearing) number of revolutions which one of the bearing rings or washers makes in relation to the other ring or washer before the first evidence of fatigue develops in the material of one of the rings or washers or one of the rolling elements

NOTE Life may also be expressed in number of hours of operation at a given constant speed of rotation.

3.2

reliability

(in the context of bearing life) for a group of apparently identical rolling bearings, operating under the same conditions, the percentage of the group that is expected to attain or exceed a specified life

NOTE The reliability of an individual rolling bearing is the probability that the bearing will attain or exceed a specified life.

3.3

rating life

predicted value of life based on a basic dynamic radial load rating or a basic dynamic axial load rating

3.4

basic rating life

rating life associated with 90 % reliability for bearings manufactured with commonly used high quality material, of good manufacturing quality, and operating under conventional operating conditions

3.5

modified rating life

rating life modified for 90 % or other reliability, bearing fatigue load, and/or special bearing properties, and/or contaminated lubricant, and/or other non-conventional operating conditions

NOTE The term "modified rating life" is new in this document and replaces "adjusted rating life".

3.6

basic dynamic radial load rating

constant stationary radial load which a rolling bearing can theoretically endure for a basic rating life of one million revolutions

NOTE In the case of a single-row angular contact bearing, the radial load rating refers to the radial component of that load which causes a purely radial displacement of the bearing rings in relation to each other.

3.7

basic dynamic axial load rating

constant centric axial load which a rolling bearing can theoretically endure for a basic rating life of one million revolutions

3.8

dynamic equivalent radial load

constant stationary radial load under the influence of which a rolling bearing would have the same life as it would attain under the actual load conditions

3.9

dynamic equivalent axial load

constant centric axial load under the influence of which a rolling bearing would have the same life as it would attain under the actual load conditions

3.10

fatigue load limit

bearing load under which the fatigue stress limit, $\sigma_{\rm u}$, is just reached in the most heavily loaded raceway contact

3.11

roller diameter

(applicable in the calculation of load ratings) theoretical diameter in a radial plane through the middle of the roller length for a symmetrical roller

NOTE 1 For a tapered roller, the applicable diameter is equal to the mean value of the diameters at the imaginary sharp corners at the large end and at the small end of the roller.

NOTE 2 For an asymmetrical convex roller, the applicable diameter is an approximation of the diameter at the point of contact between the roller and the ribless raceway at zero load.

3.12

effective roller length

 \langle applicable in the calculation of load ratings \rangle theoretical maximum length of contact between a roller and that raceway where the contact is shortest

NOTE This is normally taken to be either the distance between the theoretically sharp corners of the roller minus the roller chamfers or the raceway width, excluding the grinding undercuts, whichever is the smaller.

3.13

nominal contact angle

angle between a plane perpendicular to a bearing axis (a radial plane) and the nominal line of action of the resultant of the forces transmitted by a bearing ring or washer to a rolling element

NOTE For bearings with asymmetrical rollers, the nominal contact angle is determined by the contact with the non-ribbed raceway.

3.14

pitch diameter of ball set

diameter of the circle containing the centres of the balls in one row in a bearing

3.15

pitch diameter of roller set

diameter of the circle intersecting the roller axes at the middle of the rollers in one row in a bearing

3.16

conventional operating conditions

conditions which may be assumed to prevail for a bearing which is properly mounted and protected from foreign matter, adequately lubricated, conventionally loaded, not exposed to extreme temperature and not run at exceptionally low or high speed

3.17

viscosity ratio

actual kinematic oil viscosity at operating temperature divided by the reference kinematic viscosity for adequate lubrication

3.18

film parameter

ratio of lubricant film thickness to composite r.m.s. surface roughness, used to estimate the influence of lubrication on bearing life

3.19

pressure-viscosity coefficient

parameter characterizing the influence of oil pressure on the oil viscosity in the rolling element contact

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3.20

viscosity index

index characterizing the degree of influence of temperature on the viscosity of lubricating oils

4 Symbols

For the purposes of this document, the symbols given in ISO 15241 and the following apply.

- $a_{
 m ISO}$ life modification factor, based on a systems approach of life calculation
- a_1 life modification factor for reliability
- $b_{\rm m}$ rating factor for contemporary, commonly used, high quality hardened bearing steel in accordance with good manufacturing practices, the value of which varies with bearing type and design
- C_{a} basic dynamic axial load rating, in newtons
- $C_{\rm r}$ basic dynamic radial load rating, in newtons
- $C_{\rm u}$ fatigue load limit, in newtons
- C_{0a} basic static axial load rating³⁾, in newtons
- C_{0r} basic static radial load rating³⁾, in newtons
- D bearing outside diameter, in millimetres
- D_{pw} pitch diameter of ball or roller set, in millimetres
- $D_{\rm w}$ nominal ball diameter, in millimetres
- D_{We} roller diameter applicable in the calculation of load ratings, in millimetres
- d bearing bore diameter, in millimetres
- e limiting value of F_a/F_r for the applicability of different values of factors X and Y
- $e_{\rm C}$ contamination factor
- F_a bearing axial load (axial component of actual bearing load), in newtons
- F_r bearing radial load (radial component of actual bearing load), in newtons
- $f_{\rm c}$ factor which depends on the geometry of the bearing components, the accuracy to which the various components are made, and the material
- f_0 factor for calculation of basic static load rating³⁾
- i number of rows of rolling elements

³⁾ For definitions, calculation methods and values of C_{0a} , C_{0r} and f_0 , see ISO 76.

$L_{\sf we}$	effective roller length applicable in the calculation of load ratings, in millimetres
L ₁₀	basic rating life, in million revolutions
n	speed of rotation, in revolutions per minute
n	subscript for probability of failure, in percent
P	dynamic equivalent load, in newtons
P_{a}	dynamic equivalent axial load, in newtons
P_{r}	dynamic equivalent radial load, in newtons
S	reliability (probability of survival), in percent
X	dynamic radial load factor
Y	dynamic axial load factor
Z	number of rolling elements in a single-row bearing; number of rolling elements per row of a multi-row bearing with the same number of rolling elements per row

 κ viscosity ratio, v/v_1

nominal contact angle, in degrees

modified rating life, in million revolutions

 α

 L_{nm}

- u actual kinematic viscosity at the operating temperature, in square millimetres per second
- v_1 reference kinematic viscosity, required to obtain adequate lubrication condition, in square millimetres per second
- σ (real) stress, used in fatigue criterion, in newtons per square millimetre
- $\sigma_{\!\scriptscriptstyle \rm U}$ fatigue stress limit of raceway material, in newtons per square millimetre

5 Radial ball bearings

5.1 Basic dynamic radial load rating

5.1.1 Basic dynamic radial load rating for single bearings

The basic dynamic radial load rating for radial ball bearings is given by the equations

$$C_{\rm r} = b_{\rm m} f_{\rm c} (i\cos\alpha)^{0.7} Z^{2/3} D_{\rm w}^{1,8}$$
 (1)

for $D_{\rm w} \leqslant$ 25,4 mm

$$C_{\rm r} = 3,647 \, b_{\rm m} \, f_{\rm c} (i \cos \alpha)^{0.7} \, Z^{2/3} \, D_{\rm w}^{1.4}$$
 (2)

for
$$D_{w} > 25,4 \text{ mm}$$

where the values of $b_{\rm m}$ and $f_{\rm c}$ are given in Tables 1 and 2 respectively. They apply to bearings with a cross-sectional raceway groove radius not larger than 0,52 $D_{\rm w}$ in radial and angular contact ball bearing inner rings and not larger than 0,53 $D_{\rm w}$ in radial and angular contact ball bearing outer rings and self-aligning ball bearing inner rings.

The load-carrying ability of a bearing is not necessarily increased by the use of a smaller groove radius, but it is reduced by the use of a groove radius larger than those indicated in the previous paragraph. In the latter case, a correspondingly reduced value of f_c shall be used. Calculation of this reduced value of f_c can be carried out by means of Equation (3–15) given in ISO/TR 8646:1985.

Table 1 — Values of $b_{\rm m}$ for radial ball bearings

Bearing type	b_{m}
Radial and angular contact ball bearings (except filling slot bearings), insert bearings and self-aligning ball bearings	1,3
Filling slot bearings	1,1

5.1.2 Basic dynamic radial load rating for bearing combinations

5.1.2.1 Two single-row radial contact ball bearings operating as a unit

When calculating the basic dynamic radial load rating for two similar single-row radial contact ball bearings mounted side by side on the same shaft, such that they operate as a unit (paired mounting), the pair is considered as one double-row radial contact ball bearing.

5.1.2.2 Back-to-back and face-to-face arrangements of single-row angular contact ball bearings

When calculating the basic dynamic radial load rating for two similar single-row angular contact ball bearings mounted side by side on the same shaft, such that they operate as a unit (paired mounting) in a back-to-back or a face-to-face arrangement, the pair is considered as one double-row angular contact ball bearing.

Table 2 — Values of factor $f_{\rm C}$ for radial ball bearings

	Factor f _c							
$\frac{D_{ m W}\cos\!lpha}{D_{ m pW}}^{ m a}$	Single-row radial contact ball bearings and single- row and double-row angular contact ball bearings	Double-row radial contact ball bearings	Single-row and double-row self- aligning ball bearings	Single-row radial contact separable ball bearings (magneto bearings)				
0,01	29,1	27,5	9,9	9,4				
0,01	35,8	33,9	12,4	11,7				
0,02	40,3	38,2	14,3	13,4				
0,03	43,8	41,5	15,9	14,9				
0,05	46,7	44,2	17,3	16,2				
		_						
0,06	49,1	46,5	18,6	17,4				
0,07	51,1	48,4	19,9	18,5				
0,08	52,8	50	21,1	19,5				
0,09	54,3	51,4	22,3	20,6				
0,1	55,5	52,6	23,4	21,5				
0,11	56,6	53,6	24,5	22,5				
0,12	57,5	54,5	25,6	23,4				
0,13	58,2	55,2	26,6	24,4				
0,14	58,8	55,7	27,7	25,3				
0,15	59,3	56,1	28,7	26,2				
0,16	59,6	56,5	29,7	27,1				
0,17	59,8	56,7	30,7	27,9				
0,18	59,9	56,8	31,7	28,8				
0,19	60	56,8	32,6	29,7				
0,2	59,9	56,8	33,5	30,5				
0,21	59,8	56,6	34,4	31,3				
0,22	59,6	56,5	35,2	32,1				
0,23	59,3	56,2	36,1	32,9				
0,24	59	55,9	36,8	33,7				
0,25	58,6	55,5	37,5	34,5				
0.00	50.0	EE 4	20.2	25.0				
0,26	58,2 57.7	55,1	38,2	35,2				
0,27	57,7	54,6	38,8	35,9				
0,28	57,1 50.0	54,1	39,4	36,6				
0,29 0,3	56,6 56	53,6 53	39,9 40,3	37,2 37,8				
0,31	55,3	52,4	40,6	38,4				
0,32	54,6	51,8	40,9	38,9				
0,33	53,9	51,1	41,1	39,4				
0,34	53,2	50,4	41,2	39,8				
0,35	52,4	49,7	41,3	40,1				
0,36	51,7	48,9	41,3	40,4				
0,37	50,9	48,2	41,2	40,7				
0,38	50	47,4	41	40,8				
0,39	49,2	46,6	40,7	40,9				
0,4	48,4	45,8	40,4	40,9				

 D_{pw}

Table 3 — Values of X and Y for radial ball bearings

			Single-row bearing		Double-row bearings							
Bearin	Bearing type		e axial a, b	<i>¹</i> ⁻r	≤e	-	$\frac{F_{a}}{F_{r}} > e$	<u>-</u>	$\frac{F_{a}}{F_{r}} \leqslant e$	$\frac{F_{a}}{F_{r}} > e$		e
				X	Y	X	Y	X	Y	X	Y	
		$\frac{f_0 F_a}{C_{0r}}^c$ 0,172 0,345	$\frac{F_{a}}{iZD_{w}^{2}}$ 0,172 0,345				2,3 1,99				2,3 1,99	0,19 0,22
Radial contact ball bearings		0,689 1,03 1,38 2,07 3,45 5,17 6,89	0,689 1,03 1,38 2,07 3,45 5,17 6,89	1 0		0,56	1,71 1,55 1,45 1,31 1,15 1,04	1	0	0,56	1,71 1,55 1,45 1,31 1,15 1,04	0,26 0,28 0,3 0,34 0,38 0,42 0,42
		$\frac{f_0 i F_a}{C_{0r}}^c$	$\frac{F_{a}}{ZD_{w}^{2}}$			For th	nis type,					
	<i>α</i> = 5°	0,173 0,346 0,692 1,04 1,38 2,08 3,46 5,19 6,92	0,172 0,345 0,689 1,03 1,38 2,07 3,45 5,17 6,89	1	0	use the valuation of th	ne X, Y and les cable to e-row radial ct ball ngs.	1	2,78 2,4 2,07 1,87 1,75 1,58 1,39 1,26 1,21	0,78	3,74 3,23 2,78 2,52 2,36 2,13 1,87 1,69 1,63	0,23 0,26 0,3 0,34 0,36 0,4 0,45 0,5
Angular contact ball bearings	α = 10°	0,175 0,35 0,7 1,05 1,4 2,1 3,5 5,25	0,172 0,345 0,689 1,03 1,38 2,07 3,45 5,17 6,89	1	0	0,46	1,88 1,71 1,52 1,41 1,34 1,23 1,1 1,01	1	2,18 1,98 1,76 1,63 1,55 1,42 1,27 1,17	0,75	3,06 2,78 2,47 2,29 2,18 2 1,79 1,64 1,63	0,29 0,32 0,36 0,38 0,4 0,44 0,49 0,54
	α = 15°	0,178 0,357 0,714 1,07 1,43 2,14 3,57 5,35 7,14	0,172 0,345 0,689 1,03 1,38 2,07 3,45 5,17 6,89	1	0	0,44	1,47 1,4 1,3 1,23 1,19 1,12 1,02 1	1	1,65 1,57 1,46 1,38 1,34 1,26 1,14 1,12 1,12	0,72	2,39 2,28 2,11 2 1,93 1,82 1,66 1,63 1,63	0,38 0,4 0,43 0,46 0,47 0,5 0,55 0,56 0,56
	$\alpha = 20^{\circ}$ $\alpha = 25^{\circ}$ $\alpha = 30^{\circ}$ $\alpha = 35^{\circ}$ $\alpha = 40^{\circ}$ $\alpha = 45^{\circ}$		 - - -	1	0	0,43 0,41 0,39 0,37 0,35 0,33	1 0,87 0,76 0,66 0,57 0,5	1	1,09 0,92 0,78 0,66 0,55 0,47	0,7 0,67 0,63 0,6 0,57 0,54	1,63 1,41 1,24 1,07 0,93 0,81	0,57 0,68 0,8 0,95 1,14 1,34
Self-aligni	-	-		1	0	0,4	$0,4 \cot \alpha$	1	$0,42 \cot \alpha$	0,65	0,65 cot α	1,5 $\tan \alpha$
bearings (magneto b			1	0	0,5	2,5		_	_	—) Lise the first	0,2

Permissible maximum value depends on the bearing design (internal clearance and raceway groove depth). Use the first or second column depending on available information.

Values of X, Y and e for intermediate "relative axial loads" and/or contact angles are obtained by linear interpolation.

For values of f_0 , see ISO 76.

5.1.2.3 Tandem arrangement

The basic dynamic radial load rating, for two or more similar single-row radial contact ball bearings or two or more similar angular contact ball bearings mounted side by side on the same shaft, such that they operate as a unit (paired or stack mounting) in a tandem arrangement, is the number of bearings to the power of 0,7 times the rating of one single-row bearing. The bearings need to be properly manufactured and mounted for equal load distribution between them.

5.1.2.4 Independently replaceable bearings

If, for some technical reason, the bearing arrangement is regarded as a number of single-row specially manufactured bearings which are replaceable independently of each other, then 5.1.2.3 does not apply.

5.2 Dynamic equivalent radial load

5.2.1 Dynamic equivalent radial load for single bearings

The dynamic equivalent radial load for radial and angular contact ball bearings, under constant radial and axial loads, is given by

$$P_{\mathsf{r}} = XF_{\mathsf{r}} + YF_{\mathsf{a}} \tag{3}$$

where the values of factors *X* and *Y* are given in Table 3. These factors apply to bearings with cross-sectional groove radii according to 5.1.1. For other groove radii, calculation of *X* and *Y* can be carried out by means of 4.2 in ISO/TR 8646:1985.

5.2.2 Dynamic equivalent radial load for bearing combinations

5.2.2.1 Back-to-back and face-to-face arrangements of single-row angular contact ball bearings

When calculating the equivalent radial load for two similar single-row angular contact ball bearings mounted side by side on the same shaft, such that they operate as a unit (paired mounting) in a back-to-back or a face-to-face arrangement, the pair is considered as one double-row angular contact ball bearing.

NOTE If two similar single-row radial contact ball bearings are operating in back-to-back or face-to-face arrangement, the user should consult the bearing manufacturer about calculation of equivalent radial load.

5.2.2.2 Tandem arrangement

When calculating the equivalent radial load for two or more similar single-row radial contact ball bearings or two or more similar single-row angular contact ball bearings mounted side by side on the same shaft, such that they operate as a unit (paired or stack mounting) in a tandem arrangement, the values of X and Y for a single-row bearing shall be used.

The "relative axial load" (see Table 3) is established by using i = 1 and F_a and C_{0r} values which both refer to one of the bearings only (even though the F_r and F_a values referring to the total loads are used for the calculation of the equivalent load for the complete arrangement).

5.3 Basic rating life

5.3.1 Life equation

The basic rating life for a radial ball bearing is given by the life equation:

$$L_{10} = \left(\frac{C_{\mathsf{r}}}{P_{\mathsf{r}}}\right)^3 \tag{4}$$

The values of C_r and P_r are calculated in accordance with 5.1 and 5.2.

The life equation is also used for the evaluation of the life of two or more single-row bearings operating as a unit, as referred to in 5.1.2. In this case, the load rating $C_{\rm r}$ is calculated for the complete bearing arrangement and the equivalent load $P_{\rm r}$ is calculated for the total loads acting on the arrangement, using the values of X and Y indicated in 5.2.2.

5.3.2 Loading restriction on the life equation

The life equation gives satisfactory results for a broad range of bearing loads. However, extra-heavy loads may cause detrimental plastic deformations at the ball/raceway contacts. The user should therefore consult the bearing manufacturer to establish the applicability of the life equation in cases where $P_{\rm f}$ exceeds $C_{\rm 0r}$ or 0,5 $C_{\rm r}$, whichever is smaller.

Very light loads may cause different failure modes to occur. These failure modes are not covered by this International Standard.

6 Thrust ball bearings

6.1 Basic dynamic axial load rating

6.1.1 Basic dynamic axial load rating for single-row bearings

The basic dynamic axial load rating for single-row, single-direction or double-direction thrust ball bearings is given by

$$C_{\rm a} = b_{\rm m} f_{\rm c} Z^{2/3} D_{\rm w}^{1,8} \tag{5}$$

for $D_{\rm w} \leqslant$ 25,4 mm and α = 90°

$$C_{\rm a} = b_{\rm m} f_{\rm c} (\cos \alpha)^{0.7} \tan \alpha Z^{2/3} D_{\rm w}^{1.8}$$
 (6)

for $D_{\rm W} \leqslant$ 25,4 mm and $\alpha \neq$ 90°

$$C_a = 3.647 b_{\rm m} f_{\rm c} Z^{2/3} D_{\rm w}^{1.4}$$
 (7)

for $D_{\rm w}$ > 25,4 mm and α = 90°

$$C_{\rm a} = 3,647 \, b_{\rm m} \, f_{\rm c} \, (\cos \alpha)^{0.7} \, \tan \alpha \, Z^{2/3} \, D_{\rm w}^{-1.4}$$
 (8)

for $D_{\rm W}$ > 25,4 mm and $\alpha \neq$ 90°

where Z is the number of balls carrying load in one direction and $b_{\rm m}$ = 1,3.

Values of f_c are given in Table 4 and apply to bearings with cross-sectional raceway groove radii not larger than 0,54 D_w .

The load-carrying ability of a bearing is not necessarily increased by the use of a smaller groove radius, but is reduced by the use of a larger groove radius than that indicated above. In the latter case, a correspondingly reduced value of $f_{\rm c}$ shall be used. Calculation of this reduced value of $f_{\rm c}$ can be carried out by means of Equation (3–20) in ISO/TR 8646:1985 for bearings with $\alpha \neq 90^{\circ}$ and Equation (3–25) in ISO/TR 8646:1985 for bearings with $\alpha = 90^{\circ}$.

Table 4 — Values of f_c for thrust ball bearings

D_{W} a	$f_{\mathtt{C}}$	$D_{W} \cos\! \alpha$ a		f_{C}	
$\overline{D_{pw}}$	α = 90°	$D_{\sf pw}$	α = 45° b	α = 60°	α = 75°
0,01 0,02 0,03 0,04 0,05	36,7 45,2 51,1 55,7 59,5	0,01 0,02 0,03 0,04 0,05	42,1 51,7 58,2 63,3 67,3	39,2 48,1 54,2 58,9 62,6	37,3 45,9 51,7 56,1 59,7
0,06 0,07 0,08 0,09 0,1	62,9 65,8 68,5 71 73,3	0,06 0,07 0,08 0,09 0,1	70,7 73,5 75,9 78 79,7	65,8 68,4 70,7 72,6 74,2	62,7 65,2 67,3 69,2 70,7
0,11 0,12 0,13 0,14 0,15	75,4 77,4 79,3 81,1 82,7	0,11 0,12 0,13 0,14 0,15	81,1 82,3 83,3 84,1 84,7	75,5 76,6 77,5 78,3 78,8	
0,16 0,17 0,18 0,19 0,2	84,4 85,9 87,4 88,8 90,2	0,16 0,17 0,18 0,19 0,2	85,1 85,4 85,5 85,5 85,4	79,2 79,5 79,6 79,6 79,5	
0,21 0,22 0,23 0,24 0,25	91,5 92,8 94,1 95,3 96,4	0,21 0,22 0,23 0,24 0,25	85,2 84,9 84,5 84 83,4		
0,26 0,27 0,28 0,29 0,3	97,6 98,7 99,8 100,8 101,9	0,26 0,27 0,28 0,29 0,3	82,8 82 81,3 80,4 79,6		
0,31 0,32 0,33 0,34 0,35	102,9 103,9 104,8 105,8 106,7				

a Values of $f_{\rm C}$ for $\frac{D_{\rm W}}{D_{\rm pw}}$ or $\frac{D_{\rm W}\cos\alpha}{D_{\rm pw}}$ and/or contact angles other than those shown in the table are obtained by linear interpolation.

^b For thrust bearings $\alpha > 45^{\circ}$. Values for $\alpha = 45^{\circ}$ are given to permit interpolation of values for α between 45° and 60°.

6.1.2 Basic dynamic axial load rating for bearings with two or more rows of balls

The basic dynamic axial load rating for thrust ball bearings, with two or more rows of similar balls carrying load in the same direction, is given by

$$C_{\mathsf{a}} = \left(Z_1 + Z_2 + \dots + Z_n\right) \times \left[\left(\frac{Z_1}{C_{\mathsf{a}1}}\right)^{10/3} + \left(\frac{Z_2}{C_{\mathsf{a}2}}\right)^{10/3} + \dots + \left(\frac{Z_n}{C_{\mathsf{a}n}}\right)^{10/3} \right]^{-3/10}$$
(9)

The load ratings C_{a1} , C_{a2} , ..., C_{an} for the rows with Z_1 , Z_2 , ..., Z_n balls are calculated from the appropriate single-row bearing equation given in 6.1.1.

6.2 Dynamic equivalent axial load

The dynamic equivalent axial load for thrust ball bearings with $\alpha \neq 90^{\circ}$, under constant radial and axial loads, is given by

$$P_{\mathbf{a}} = XF_{\mathbf{r}} + YF_{\mathbf{a}} \tag{10}$$

where the values of X and Y are given in Table 5. These factors apply to bearings with cross-sectional groove radii according to 6.1.1. For other groove radii, calculation of X and Y can be carried out by means of 4.2 in ISO/TR 8646:1985.

Thrust ball bearings with $\alpha = 90^{\circ}$ can support axial loads only. The dynamic equivalent axial load for this type of bearing is given by

$$P_{\mathbf{a}} = F_{\mathbf{a}} \tag{11}$$

Table 5 —	- Values d	f Y and	V for thrust	ball bearings
I able 5 —	- vaiues i	JI A AIIU	/ IOI HIIUSE	Dali Dealiilus

	Single-direction bearings ^b		D				
αª	$\frac{F_{a}}{F_{r}} > e$		$\frac{F_{a}}{F_{r}} \leqslant e$		$\frac{F_{a}}{F_{r}} > e$		e
	X	Y	X	Y	X	Y	
45° c 50° 55° 60° 65° 70° 75° 80° 85°	0,66 0,73 0,81 0,92 1,06 1,28 1,66 2,43 4,8	1	1,18 1,37 1,6 1,9 2,3 2,9 3,89 5,86 11,75	0,59 0,57 0,56 0,55 0,54 0,53 0,52 0,52 0,52	0,66 0,73 0,81 0,92 1,06 1,28 1,66 2,43 4,8	1	1,25 1,49 1,79 2,17 2,68 3,43 4,67 7,09 14,29
<i>α</i> ≠ 90°	$1,25\tan\alpha\left(1-\frac{2}{3}\sin\alpha\right)$	1	$\frac{20}{13}\tan\alpha\left(1-\frac{1}{3}\sin\alpha\right)$	$\frac{10}{13} \left(1 - \frac{1}{3} \sin \alpha \right)$	$1,25\tan\alpha\left(1-\frac{2}{3}\sin\alpha\right)$	1	1,25 tanα

Values of X, Y and e for intermediate values of α are obtained by linear interpolation.

b $\frac{F_{\mathbf{a}}}{F_{\mathbf{r}}} \le e$ is unsuitable for single-direction bearings.

For thrust bearings, $\alpha > 45^{\circ}$. Values for $\alpha = 45^{\circ}$ are given to permit interpolation of values for α between 45° and 50° .

6.3 Basic rating life

6.3.1 Life equation

The basic rating life for a thrust ball bearing is given by the life equation:

$$L_{10} = \left(\frac{C_{\mathsf{a}}}{P_{\mathsf{a}}}\right)^3 \tag{12}$$

The values of C_a and P_a are calculated in accordance with 6.1 and 6.2.

6.3.2 Loading restriction on the life equation

The life equation gives satisfactory results for a broad range of bearing loads. However, extra-heavy loads may cause detrimental plastic deformations at the ball/raceway contacts. The user should therefore consult the bearing manufacturer to establish the applicability of the life equation in cases where P_a exceeds 0,5 C_a .

Very light loads may cause different failure modes to occur. These failure modes are not covered by this International Standard.

7 Radial roller bearings

7.1 Basic dynamic radial load rating

7.1.1 Basic dynamic radial load rating for single bearings

The basic dynamic radial load rating for a radial roller bearing is given by

$$C_{\rm r} = b_{\rm m} f_{\rm c} \left(i L_{\rm we} \cos \alpha \right)^{7/9} Z^{3/4} D_{\rm we}^{29/27}$$
 (13)

where the values of $b_{\rm m}$ and $f_{\rm c}$ are given in Tables 6 and 7 respectively. They are maximum values applicable only to roller bearings in which, under a bearing load, the contact stress is substantially uniform along the most heavily loaded roller/raceway contact.

Smaller values of $f_{\rm C}$ than those given in Table 7 should be used if, under load, an accentuated stress concentration is present in some part of the roller/raceway contact. Such stress concentrations are to be expected, at the centre of the nominal contact points and at the extremities of the line contacts, in bearings where the rollers are not accurately guided and in bearings having rollers longer than 2,5 times their diameter.

Table 6 — Values of $b_{\rm m}$ for radial roller bearings

Bearing type	b_{m}
Cylindrical roller bearings, tapered roller bearings and needle roller bearings with machined rings	1,1
Drawn cup needle roller bearings	1
Spherical roller bearings	1,15

Table 7 — Maximum values of $f_{\rm C}$ for radial roller bearings

$\frac{D_{we} \cos\!lpha}{D_{pw}}^{a}$	fс
0,01	52,1
0,02	60,8
0,03	66,5
0,04	70,7
0,05	74,1
0,06	76,9
0,07	79,2
0,08	81,2
0,09	82,8
0,1	84,2
0,11	85,4
0,12	86,4
0,13	87,1
0,14	87,7
0,15	88,2
0,16	88,5
0,17	88,7
0,18	88,8
0,19	88,8
0,2	88,7
0,21	88,5
0,22	88,2
0,23	87,9
0,24	87,5
0,25	87
0,26	86,4
0,27	85,8
0,28	85,2
0,29	84,5
0,3	83,8
	$\frac{\cos lpha}{D_{ m pw}}$ are obtained by linear interpolation.

^{· ·}

7.1.2 Basic dynamic radial load rating for bearing combinations

7.1.2.1 Back-to-back and face-to-face arrangements

When calculating the basic dynamic radial load rating for two similar single-row radial roller bearings mounted side by side on the same shaft, such that they operate as a unit (paired mounting) in a back-to-back or a face-to-face arrangement, the pair is considered as one double-row bearing.

7.1.2.2 Independently replaceable bearings in back-to-back and face-to-face arrangements

If, for some technical reason, the bearing arrangement is regarded as two bearings which are replaceable independently of each other, then 7.1.2.1 does not apply.

7.1.2.3 Tandem arrangement

The basic dynamic radial load rating for two or more similar single-row radial roller bearings mounted side by side on the same shaft, such that they operate as a unit (paired or stack mounting) in tandem arrangement, is the number of bearings to the power of 7/9, times the rating of one single-row bearing. The bearings need to be properly manufactured and mounted for equal load distribution of the load between them.

7.1.2.4 Independently replaceable bearings in tandem arrangement

If, for some technical reason, the bearing arrangement is regarded as the number of single-row bearings which are replaceable independently of each other, then 7.1.2.3 does not apply.

7.2 Dynamic equivalent radial load

7.2.1 Dynamic equivalent radial load for single bearings

The dynamic equivalent radial load for radial roller bearings with $\alpha \neq 0^{\circ}$, under constant radial and axial loads, is given by

$$P_{\mathsf{r}} = X F_{\mathsf{r}} + Y F_{\mathsf{a}} \tag{14}$$

where the values of X and Y are given in Table 8.

The dynamic equivalent radial load for radial roller bearings with $\alpha = 0^{\circ}$, and subjected to radial load only, is given by

$$P_{\mathbf{r}} = F_{\mathbf{r}} \tag{15}$$

NOTE The ability of radial roller bearings with $\alpha = 0^{\circ}$ to support axial loads varies considerably with bearing design and execution. The bearing user should therefore consult the bearing manufacturer for recommendations regarding the evaluation of equivalent load and life for cases where bearings with $\alpha = 0^{\circ}$ are subjected to axial load.

7.2.2 Dynamic equivalent radial load for bearing combinations

7.2.2.1 Back-to-back and face-to-face arrangements of single-row angular contact roller bearings

When calculating the equivalent radial load for two similar single-row angular contact roller bearings mounted side by side on the same shaft, such that they operate as a unit (paired mounting) in a back-to-back or a face-to-face arrangement, and which, according to 7.1.2.1, is considered as one double-row bearing, the values of *X* and *Y* for double-row bearings given in Table 8 shall be used.

7.2.2.2 Tandem arrangement

When calculating the equivalent radial load rating for two or more similar single-row angular contact roller bearings mounted side by side on the same shaft, such that they operate as a unit (paired or stack mounting) in a tandem arrangement, the *X* and *Y* factors for single-row bearings given in Table 8 shall be used.

Table 8 — Values of X and Y for radial roller bearings

Bearing type		$\frac{F_{a}}{F_{r}} \leqslant e$	-	$\frac{F_a}{F_r} > e$	e	
	X	Y	X	Y		
Single-row, $\alpha \neq 0^{\circ}$	1	0	0,4	$0.4 \cot \alpha$	1,5 $ anlpha$	
Double-row, $\alpha \neq 0^{\circ}$	1	0,45 cotα	0,67	$0,67\cot\alpha$	1,5 $\tan lpha$	

7.3 Basic rating life

7.3.1 Life equation

The basic rating life for a radial roller bearing is given by the life equation:

$$L_{10} = \left(\frac{C_{\rm r}}{P_{\rm r}}\right)^{10/3} \tag{16}$$

The values of C_r and P_r are calculated in accordance with 7.1 and 7.2.

This life equation is also used for the evaluation of the life of two or more single-row bearings operating as a unit, as referred to in 7.1.2. In this case, the load rating $C_{\rm r}$ is calculated for the complete bearing arrangement and the equivalent load $P_{\rm r}$ is calculated for the total loads acting on the arrangement, using the values of X and Y indicated in 7.2.2.

7.3.2 Loading restriction on the life equation

The life equation gives satisfactory results for a broad range of bearing loads. However, extra-heavy loads may cause accentuated stress concentrations in some part of the roller/raceway contacts. The user should therefore consult the bearing manufacturer to establish the applicability of the life equation in cases where $P_{\rm r}$ exceeds 0,5 $C_{\rm r}$.

Very light loads may cause different failure modes to occur. These failure modes are not covered by this International Standard.

8 Thrust roller bearings

8.1 Basic dynamic axial load rating

8.1.1 Basic dynamic axial load rating for single-row bearings

A thrust roller bearing is considered as a single-row bearing only if all rollers carrying load in the same direction contact the same washer raceway area.

The basic dynamic axial load rating for single-row, single-direction or double-direction thrust roller bearing is given by

$$C_{\rm a} = b_{\rm m} f_{\rm c} L_{\rm we}^{7/9} Z^{3/4} D_{\rm we}^{29/27} \tag{17}$$

for $\alpha = 90^{\circ}$

$$C_{\rm a} = b_{\rm m} f_{\rm c} \left(L_{\rm we} \cos \alpha \right)^{7/9} \tan \alpha Z^{3/4} D_{\rm we}^{29/27}$$
 (18)

for $\alpha \neq 90^{\circ}$

where Z is the number of rollers carrying load in one direction.

If several rollers, on the same side of the bearing axis, are located with their axes coinciding, these rollers are considered as one roller with a length L_{we} equal to the sum of the lengths (see 3.12) of the several rollers.

Values of $b_{\rm m}$ and values of $f_{\rm c}$ are given in Tables 9 and 10 respectively. They are maximum values, only applicable to roller bearings in which, under a bearing load, the contact stress is substantially uniform along the most heavily loaded roller/raceway contact.

Smaller values of $f_{\rm C}$ than those given in Table 10 should be used if, under load, an accentuated stress concentration is present in some part of the roller/raceway contact. Such stress concentrations must be expected, for example, at the centre of nominal point contacts, at the extremities of line contacts, in bearings where the rollers are not accurately guided and in bearings with rollers longer than 2,5 times the roller diameter.

Smaller values of $f_{\rm C}$ should also be considered for thrust roller bearings in which the geometry causes excessive slip in the roller/raceway contact areas, for example bearings with cylindrical rollers which are long in relation to the pitch diameter of the roller set.

Table 9 — Values of $b_{\rm m}$ for thrust roller bearings

Bearing type	b_{m}
Cylindrical roller bearings and needle roller bearings	1
Tapered roller bearings	1,1
Spherical roller bearings	1,15

Table 10 — Maximum values of $f_{\rm C}$ for thrust roller bearings

Values of $f_{\rm C}$ for intermediate values of $\frac{D_{\rm We}}{D_{\rm pw}}$ or $\frac{D_{\rm We}\cos\alpha}{D_{\rm pw}}$ are obtained by linear interpolation.

8.1.2 Basic dynamic axial load rating for bearings with two or more rows of rollers

The basic dynamic axial load rating for thrust roller bearings with two or more rows of rollers carrying load in the same direction is given by

$$C_{a} = \left(Z_{1}L_{\text{we1}} + Z_{2}L_{\text{we2}} + \ldots + Z_{n}L_{\text{wen}}\right) \times \left[\left(\frac{Z_{1}L_{\text{we1}}}{C_{\text{a1}}}\right)^{9/2} + \left(\frac{Z_{2}L_{\text{we2}}}{C_{\text{a2}}}\right)^{9/2} + \ldots + \left(\frac{Z_{n}L_{\text{wen}}}{C_{\text{an}}}\right)^{9/2}\right]^{-2/9}$$
(19)

The load ratings $C_{a1}, C_{a2}, \ldots, C_{an}$ for the rows with Z_1, Z_2, \ldots, Z_n rollers of lengths $L_{we1}, L_{we2}, \ldots, L_{wen}$ are calculated from the appropriate single-row bearing equation given in 8.1.1.

Rollers and/or portions of rollers which contact the same washer raceway area belong to one row.

b Applicable for $45^{\circ} < \alpha < 60^{\circ}$.

^c Applicable for $60^{\circ} \le \alpha < 75^{\circ}$.

d Applicable for $75^{\circ} \leqslant \alpha < 90^{\circ}$.

8.1.3 Basic dynamic axial load rating for bearing combinations

8.1.3.1 Tandem arrangement

The basic dynamic axial load rating for two or more similar single-direction thrust roller bearings mounted side by side on the same shaft, such that they operate as a unit (paired or stack mounting) in tandem arrangement, is the number of bearings to the power of 7/9, times the rating of one bearing. The bearings need to be properly manufactured and mounted for equal load distribution of the load between them.

8.1.3.2 Independently replaceable bearings

If, for some technical reason, the bearing arrangement is regarded as the number of single-direction bearings which are replaceable independently of each other, then 8.1.3.1 does not apply.

8.2 Dynamic equivalent axial load

The dynamic equivalent axial load for thrust roller bearings with $\alpha \neq 90^{\circ}$, under constant radial and axial loads, is given by

$$P_{\mathbf{a}} = X F_{\mathbf{r}} + Y F_{\mathbf{a}} \tag{20}$$

where the values of *X* and *Y* are given in Table 11.

Thrust roller bearings with $\alpha = 90^{\circ}$ can support axial loads only. The dynamic equivalent axial load for this type of bearing is given by

$$P_{\mathbf{a}} = F_{\mathbf{a}} \tag{21}$$

Table 11 — Values of X and Y for thrust roller bearings

Bearing type	$\frac{F_{a}}{F_{r}} \leqslant e$		$\frac{F_{a}}{F_{r}} > e$		e	
	X	Y	X	Y		
Single-direction, $\alpha \neq 90^{\circ}$	a	a	anlpha	1	1,5 $ anlpha$	
Double-direction, $\alpha \neq 90^{\circ}$	1,5 $\tan \alpha$	0,67	tan lpha	1	1,5 $ anlpha$	
a $\frac{F_{\mathbf{a}}}{F_{\mathbf{f}}} \leqslant e$ is unsuitable for single-direction bearings.						

8.3 Basic rating life

8.3.1 Life equation

The basic rating life for a thrust roller bearing is given by the life equation:

$$L_{10} = \left(\frac{C_{\rm a}}{P_{\rm a}}\right)^{10/3} \tag{22}$$

The values of C_a and P_a are calculated in accordance with 8.1 and 8.2.

This life equation is also used for the evaluation of the life of two or more single-direction thrust roller bearings operating as a unit, as referred to in 8.1.3. In this case, the load rating $C_{\rm a}$ is calculated for the complete bearing arrangement and the equivalent load $P_{\rm a}$ is calculated for the total loads acting on the arrangement, using the values of X and Y given for single-direction bearings in 8.2.

8.3.2 Loading restriction on the life equation

The life equation gives satisfactory results for a broad range of bearing loads. However, extra-heavy loads may cause accentuated stress concentrations in some part of the roller/raceway contacts. The user should therefore consult the bearing manufacturer to establish the applicability of the life equation in cases where $P_{\rm a}$ exceeds 0,5 $C_{\rm a}$.

Very light loads may cause different failure modes to occur. These failure modes are not covered by this International Standard.

9 Modified rating life

9.1 General

For many years, the use of the basic rating life L_{10} as a criterion of bearing performance has proved satisfactory. This life is associated with 90 % reliability, with commonly used high quality material, good manufacturing quality, and with conventional operating conditions.

However, for many applications it has become desirable to calculate the life for a different level of reliability and/or for a more accurate life calculation under specified lubrication and contamination conditions. With modern high quality bearing steel, it has been found that, under favourable operating conditions and below certain Hertzian rolling element contact stress, very long bearing lives, compared with the L_{10} life, can be obtained if the fatigue limit of the bearing steel is not exceeded. On the other hand, bearing lives shorter than the L_{10} life can be obtained under unfavourable operating conditions.

A systems approach to the fatigue life calculation has been used in this International Standard. With such a method, the influence on the life of the system due to variation and interaction of interdependent factors is considered by referring all influences to the additional stress they give rise to in the rolling element contacts and under the contact regions.

In this International Standard, a life modification factor, $a_{\rm ISO}$, is introduced, based on a systems approach of life calculation in addition to the modification factor $a_{\rm 1}$. These factors are applied in the modified rating life equation

$$L_{nm} = a_1 a_{ISO} L_{10} (23)$$

The modification factor for reliability, a_1 , for a range of reliability values is given in 9.2 and the method for evaluating the modification factor for systems approach, $a_{\rm ISO}$, is detailed in 9.3.

9.2 Life modification factor for reliability

Reliability is defined in 3.2. The modified rating life is calculated according to Equation (23) and values of the life modification factor for reliability, a_1 , are given in Table 12.

NOTE In Table 12, the a_1 values for the reliabilities 95 % to 99 % have been modified slightly compared with the corresponding values in the previous edition of this International Standard.

Table 12 — Life modification factor for reliability, a_1

Reliability	I	a_1	
%	L_{nm}		
90	L_{10m}	1	
95	L_{5m}	0,64	
96	L_{4m}	0,55	
97	L_{3m}	0,47	
98	L_{2m}	0,37	
99	L_{1m}	0,25	
99,2	$L_{0,8m}$	0,22	
99,4	$L_{\sf 0,6m}$	0,19	
99,6	$L_{0,4m}$	0,16	
99,8	$L_{0,2m}$	0,12	
99,9	$L_{0,1m}$	0,093	
99,92	$L_{ m 0,08m}$	0,087	
99,94	$L_{ m 0,06m}$	0,080	
99,95	$L_{ m 0,05m}$	0,077	

9.3 Life modification factor for systems approach

9.3.1 General

Below a certain load, a modern high quality bearing can attain an infinite life, if the lubrication conditions, the cleanliness and other operating conditions are favourable.

For rolling bearings of commonly used high quality material and good manufacturing quality, the fatigue stress limit is reached at a contact stress of approximately 1 500 MPa. This stress value takes into account additional stresses occurring due to manufacturing tolerances and operating conditions. Reduced manufacturing accuracy and/or material quality result in a lower fatigue stress limit.

In many applications, contact stresses are, however, larger than 1 500 MPa and, in addition, the operating conditions can give rise to additional stresses and by that further reduce the bearing life.

It is possible to relate all operational influences to the applied stresses and to the strength of the material, e.g.:

- indentations give rise to edge stresses;
- a thin oil film increases the stresses in the contact region between raceway and rolling element;
- an increased temperature reduces the fatigue stress limit of the material, i.e. its strength;
- a tight inner ring fit gives rise to hoop stresses.

The different influences on bearing life are dependent on each other. A systems approach of the fatigue life calculation is therefore appropriate, as the influence on the life of the system from variation and interaction of interdependent factors will then be considered. For performing modified life systems approach calculations, practical methods have been developed for determining the life modification factor, $a_{\rm ISO}$, which consider the fatigue stress limit of the bearing steel and make it easy to estimate the influence of lubrication and contamination on bearing life, see 9.3.3.

A theoretical explanation on how to include the additional influences of radial clearance in an operating bearing, and non-uniform raceway compressive stresses from bearing misalignment, is presented in ISO/TS 16281 [1]

9.3.2 Fatigue load limit

The life modification factor, $a_{\rm ISO}$, can be expressed as a function of $\sigma_{\rm u}/\sigma$, the fatigue stress limit divided by the real stress with as many influencing factors as possible considered (see Figure 1).

In Figure 1, the diagram for a given lubrication condition also illustrates how $a_{\rm ISO}$ asymptotically approaches infinity, if the real stress, σ , is decreased down to the fatigue stress limit, $\sigma_{\rm U}$, when a fatigue criterion applies. Traditionally, the orthogonal shear stress has been used as the fatigue criterion in bearing life calculations (see Reference [3] in the Bibliography). The diagram in Figure 1 can therefore also be based on fatigue strength in shear.

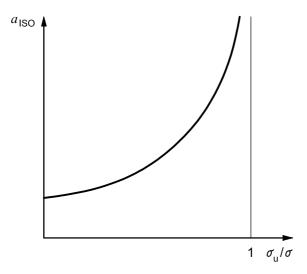


Figure 1 — Life modification factor, a_{ISO}

The diagram in Figure 1 can be expressed with the following equation:

$$a_{\rm ISO} = f\left(\frac{\sigma_{\rm u}}{\sigma}\right) \tag{24}$$

The fatigue determining stress in the raceway is mainly dependent on the bearing internal load distribution and the distribution of subsurface-stresses in the most heavily loaded contact. To facilitate the practical calculation a fatigue load limit, $C_{\rm u}$, is introduced (see Reference [3]).

In analogy to the static load rating in ISO 76, $C_{\rm u}$ is defined as the load at which the fatigue stress limit, $\sigma_{\rm u}$, is just reached in the most heavily loaded raceway contact. The ratio $\frac{\sigma_{\rm u}}{\sigma}$ can then be sufficiently approximated by the ratio $\frac{C_{\rm u}}{P}$, and the life modification factor, $a_{\rm ISO}$, expressed as

$$a_{\rm ISO} = f\left(\frac{C_{\rm u}}{P}\right) \tag{25}$$

In the calculation of $C_{\rm u}$, the following influences have to be considered:

- the type, size and internal geometry of the bearing;
- the profile of rolling elements and raceways;
- the manufacturing quality;
- the fatigue limit of the raceway material.

Values of the fatigue load limit, C_{II} , can be determined by means of the equations in Annex B.

9.3.3 Practical methods for estimating the life modification factor

9.3.3.1 General

Modern technology makes it possible to determine $a_{\rm ISO}$ by combining computer supported theory with empirical tests and practical experience. Besides bearing type, fatigue load and bearing load, the factor $a_{\rm ISO}$ in this International Standard considers the influence of:

- lubrication (type of lubricant, viscosity, bearing speed, bearing size, additives);
- environment (contamination level, seals);
- contaminant particles (hardness and particle size in relation to bearing size, lubrication method, filtration);
- mounting (cleanliness during mounting, e.g. by careful flushing and filtering of supplied oil).

The influence of bearing clearance and misalignment on bearing life is given in ISO/TS 16281 [1].

The life modification factor, $a_{\rm ISO}$, can be derived from the following equation:

$$a_{\rm ISO} = f\left(\frac{e_{\rm C} C_{\rm U}}{P}, \kappa\right)$$
 (26)

The factors $e_{\rm C}$ and κ take into consideration contamination and lubricating condition. They are dealt with in 9.3.3.2 and 9.3.3.3.

Values for the life modification factor $a_{\rm ISO}$ can be taken from Figures 3 to 6 for the respective bearing type.

P is the dynamic equivalent load according to Equations (3), (10), (11), (14), (15), (20) and (21).

9.3.3.2 Contamination factor

When the lubricant is contaminated with solid particles, permanent indentations in the raceway can be generated when these particles are over rolled. At these indentations, local stress risers are generated, which will lead to a reduced life of the rolling bearing. This life reduction due to contamination in the lubricant film is taken into account by the contamination factor, $e_{\rm C}$.

The life reduction caused by solid particles in the lubricant film is dependent on:

- type, size, hardness and quantity of the particles;
- lubricant film thickness (viscosity ratio, κ , see 9.3.3.3);
- bearing size.

Guide values for the contamination factor can be taken from Table 13, which shows typical levels of contamination for well lubricated bearings. More accurate and detailed guide values can be obtained from the diagrams or equations in Annex A. These values are valid for a mixture of particles of different hardness and toughness in which the hard particles determine the modified rating life. If large hard particles exist, beyond the expected sizes in the cleanliness classes of ISO 4406^[7], the bearing life can be considerably shorter than the calculated rating life.

Table 13 — Contamination factor, $e_{\rm C}$

Level of contamination	e_{C}		
Level of contamination	D_{pw} < 100 mm	$D_{pw} \geqslant$ 100 mm	
Extreme cleanliness			
Particle size of the order of lubricant film thickness; laboratory conditions	1	1	
High cleanliness			
Oil filtered through extremely fine filter; conditions typical of bearing greased for life and sealed	0,8 to 0,6	0,9 to 0,8	
Normal cleanliness			
Oil filtered through fine filter; conditions typical of bearings greased for life and shielded	0,6 to 0,5	0,8 to 0,6	
Slight contamination	0.5 to 0.3	0.6 to 0.4	
Slight contamination in lubricant	0,5 to 0,5	0,0 to 0,4	
Typical contamination		0,4 to 0,2	
Conditions typical of bearings without integral seals; course filtering; wear particles and ingress from surroundings	0,3 to 0,1		
Severe contamination		0,1 to 0	
Bearing environment heavily contaminated and bearing arrangement with inadequate sealing	0,1 to 0		
Very severe contamination	0	0	

Contamination by water or other fluids is not considered in this International Standard.

In the case of severe contamination ($e_C \rightarrow 0$), failure may be caused by wear, and the life of the bearing can be far below a calculated modified rating life.

9.3.3.3 Viscosity ratio

9.3.3.3.1 Calculation of viscosity ratio

The effectiveness of the lubricant is primarily determined by the degree of surface separation between the rolling contact surfaces. If an adequate lubricant separation film is to be formed, the lubricant must have a given minimum viscosity when the application has reached its operating temperature. The condition of the lubricant separation is described by the viscosity ratio, κ , as the ratio of the actual kinematic viscosity, ν , to the reference kinematic viscosity, ν_1 . The kinematic viscosity, ν , is considered when the lubricant is at operating temperature.

$$\kappa = \frac{v}{v_1} \tag{27}$$

In order to form an adequate lubricant film between the rolling contact surfaces, the lubricant must retain a certain minimum viscosity when the lubricant is at operating temperature. The bearing life may be extended by increasing the operating viscosity ν .

The reference kinematic viscosity, ν_1 , can be estimated by means of the diagram in Figure 2, depending on bearing speed and pitch diameter, D_{pw} , [the mean bearing diameter 0,5 (d+D) can also be used] or be calculated with the following Equations (28) and (29):

$$v_1 = 45\,000\,n^{-0.83}\,D_{\rm pw}^{-0.5}$$
 for $n < 1\,000$ r/min (28)

$$v_1 = 4500 \, n^{-0.5} \, D_{\text{DW}}^{-0.5}$$
 for $n \ge 1000 \, \text{r/min}$ (29)

9.3.3.3.2 Restriction of the calculation of the viscosity ratio

The calculation of κ is based on mineral oils and on bearing raceway surfaces machined with good manufacturing quality.

The diagram in Figure 2 and Equations (28) and (29) can also be used approximately for synthetic oils of, e.g. synthetic hydrocarbon (SHC) type, for which the larger viscosity index (less change of viscosity with temperature) is compensated for by a larger pressure-viscosity coefficient for mineral oils, and by that about the same oil film is built up at different operating temperatures if both oil types have the same viscosity at 40 °C.

If, however, there is a need of a more detailed estimation of the κ value, e.g. for especially machined raceway surface finish, specific pressure-viscosity coefficient, specific density, etc., the film parameter, Λ , can be applied. This film parameter is well known in literature, e.g. in Reference [4].

When Λ is calculated, the κ value can be approximately estimated with the following equation:

$$\kappa \approx \Lambda^{1,3}$$

9.3.3.3.3 Grease lubrication

The diagram in Figure 2 and Equations (28) and (29) apply equally to the base oil viscosity of greases. With grease lubrication, the contacts may be operating in a severely starved condition because of the poor bleeding capability of the grease leading to poor lubrication and possible reduction of life.

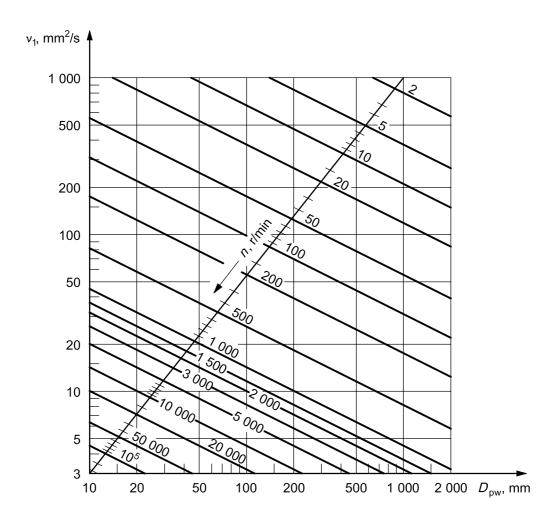


Figure 2 — Reference kinematic viscosity, v_1

9.3.3.3.4 Consideration of EP additives

In case of a viscosity ratio $\kappa < 1$ and a contamination factor $e_{\rm C} \geqslant 0.2$ for this viscosity ratio, a value of $\kappa = 1$ can be used in the calculation of $e_{\rm C}$ and $a_{\rm ISO}$ if a lubricant with proven effective EP additives is used. In this case, the life modification factor, $a_{\rm ISO}$, shall be limited to $a_{\rm ISO} \leqslant 3$, respectively to the life modification factor, $a_{\rm ISO}$, calculated for normal lubricants with the actual κ value, if this $a_{\rm ISO}$ value is above 3.

This motivation for increasing the κ value is that a favourable smoothening effect of the contacting surfaces can be expected when an effective EP additive is used. In the case of severe contamination (contamination factor $e_{\rm C} < 0.2$), the efficiency of the EP additives shall be proven under actual lubricant contamination. The efficiency of the EP additives should be proven in the actual application or in an appropriate bearing test.

9.3.3.4 Calculation of the life modification factor

The life modification factor, $a_{\rm ISO}$, can be estimated easily by means of Figures 3, 4, 5, and 6 or calculated with Equations (31) to (42). How the factors in the diagrams and equations, $C_{\rm u}$, $e_{\rm C}$ and κ , can be determined is shown in 9.3.2, 9.3.3.2 and 9.3.3.3.

Guide values for the contamination factor, $e_{\rm C}$, can be taken from Table 13. More accurate and detailed guide values can be obtained from the diagrams or equations in Annex A.

For practical considerations, the life modification factor shall be limited to $a_{\rm ISO} \leqslant$ 50. This limit also applies when $\frac{e_{\rm C} \, C_{\rm U}}{P} >$ 5.

For κ values > 4, the value κ = 4 shall be used.

When the κ value is < 0,1, calculation of the $a_{\rm ISO}$ factor is not possible with currently accepted experience and $a_{\rm ISO}$ values for κ < 0,1 are out of range of equations and diagrams.

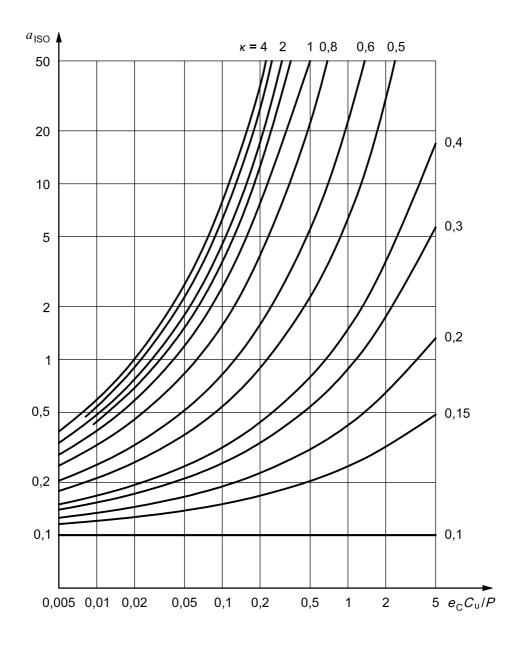


Figure 3 — Life modification factor, $a_{\rm ISO}$, for radial ball bearings

The curves in Figure 3 are based on the following equations:

$$a_{\rm ISO} = 0.1 \left[1 - \left(2,5671 - \frac{2,2649}{\kappa^{0,054381}} \right)^{0.83} \left(\frac{e_{\rm C} C_{\rm u}}{P} \right)^{1/3} \right]^{-9.3}$$
 for $0.1 \le \kappa < 0.4$ (31)

$$a_{\rm ISO} = 0.1 \left[1 - \left(2,5671 - \frac{1,9987}{\kappa^{0,19087}} \right)^{0.83} \left(\frac{e_{\rm C} C_{\rm u}}{P} \right)^{1/3} \right]^{-9.3}$$
 for $0.4 \le \kappa < 1$ (32)

$$a_{\rm ISO} = 0.1 \left[1 - \left(2,5671 - \frac{1,9987}{\kappa^{0.071739}} \right)^{0.83} \left(\frac{e_{\rm C} C_{\rm u}}{P} \right)^{1/3} \right]^{-9.3}$$
 for $1 \le \kappa \le 4$ (33)

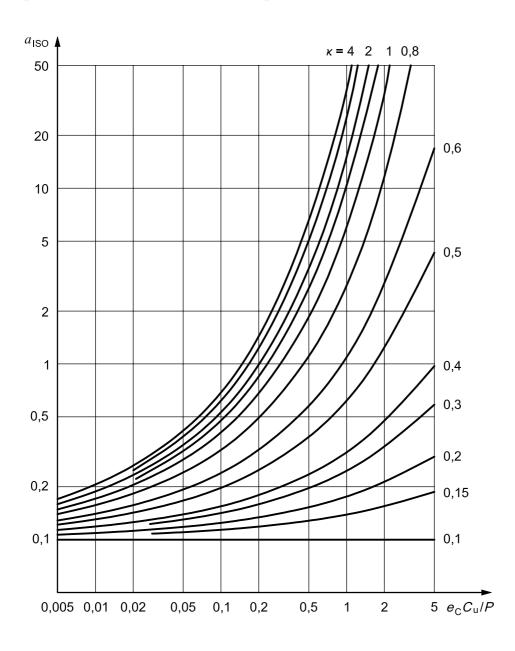


Figure 4 — Life modification factor, a_{ISO} , for radial roller bearings

The curves in Figure 4 are based on the following equations:

$$a_{\rm ISO} = 0.1 \left[1 - \left(1,5859 - \frac{1,3993}{\kappa^{0,054381}} \right) \left(\frac{e_{\rm C} C_{\rm u}}{P} \right)^{0.4} \right]^{-9,185}$$
 for $0.1 \le \kappa < 0.4$ (34)

$$a_{\rm ISO} = 0.1 \left[1 - \left(1,5859 - \frac{1,2348}{\kappa^{0,19087}} \right) \left(\frac{e_{\rm C} C_{\rm u}}{P} \right)^{0.4} \right]^{-9,185}$$
 for $0.4 \le \kappa < 1$ (35)

$$a_{\rm ISO} = 0.1 \left[1 - \left(1,5859 - \frac{1,2348}{\kappa^{0,071739}} \right) \left(\frac{e_{\rm C} C_{\rm U}}{P} \right)^{0.4} \right]^{-9,185}$$
 for $1 \le \kappa \le 4$

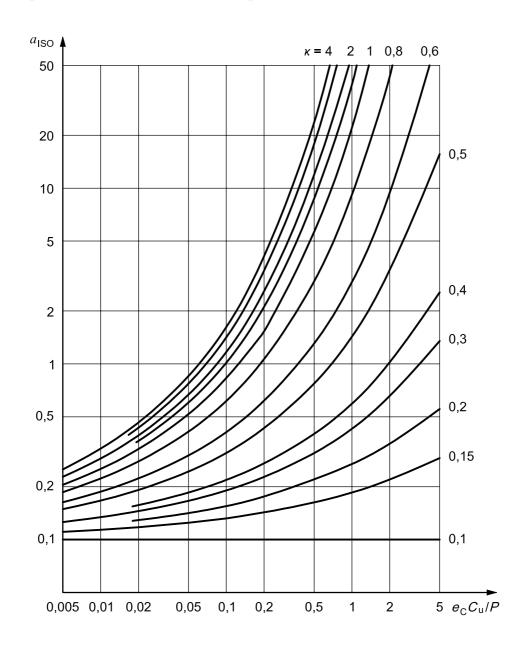


Figure 5 — Life modification factor, $a_{\rm ISO}$, for thrust ball bearings

The curves in Figure 5 are based on the following equations:

$$a_{\rm ISO} = 0.1 \left[1 - \left(2,5671 - \frac{2,2649}{\kappa^{0,054381}} \right)^{0.83} \left(\frac{e_{\rm C} C_{\rm u}}{3P} \right)^{1/3} \right]^{-9.3} \quad \text{for } 0.1 \leqslant \kappa < 0.4$$
 (37)

$$a_{\rm ISO} = 0.1 \left[1 - \left(2,5671 - \frac{1,9987}{\kappa^{0,19087}} \right)^{0.83} \left(\frac{e_{\rm C} C_{\rm U}}{3P} \right)^{1/3} \right]^{-9.3}$$
 for $0.4 \le \kappa < 1$ (38)

$$a_{\rm ISO} = 0.1 \left[1 - \left(2,5671 - \frac{1,9987}{\kappa^{0.071739}} \right)^{0.83} \left(\frac{e_{\rm C} C_{\rm u}}{3P} \right)^{1/3} \right]^{-9.3}$$
 for $1 \le \kappa \le 4$ (39)

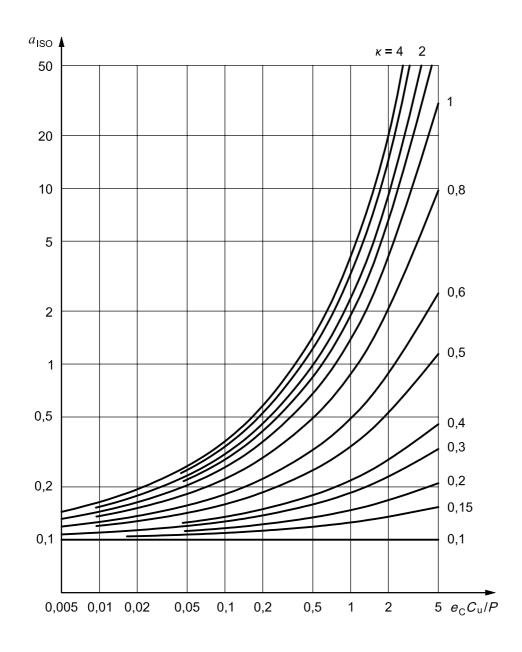


Figure 6 — Life modification factor, a_{ISO} , for thrust roller bearings

The curves in Figure 6 are based on the following equations:

$$a_{\rm ISO} = 0.1 \left[1 - \left(1,5859 - \frac{1,3993}{\kappa^{0,054381}} \right) \left(\frac{e_{\rm C} C_{\rm U}}{2,5 P} \right)^{0.4} \right]^{-9,185}$$
 for $0.1 \le \kappa < 0.4$ (40)

$$a_{\rm ISO} = 0.1 \left[1 - \left(1,5859 - \frac{1,2348}{\kappa^{0,19087}} \right) \left(\frac{e_{\rm C} C_{\rm u}}{2,5P} \right)^{0.4} \right]^{-9,185}$$
 for $0.4 \le \kappa < 1$ (41)

$$a_{\rm ISO} = 0.1 \left[1 - \left(1,5859 - \frac{1,2348}{\kappa^{0,071739}} \right) \left(\frac{e_{\rm C} C_{\rm U}}{2,5P} \right)^{0.4} \right]^{-9,185}$$
 for $1 \le \kappa \le 4$

Annex A

(informative)

Detailed method for estimating the contamination factor

A.1 General

A simplified method to estimate the size of the contamination factor, $e_{\rm C}$, is given in 9.3.3.2. This annex provides a more advanced and detailed method to calculate the $e_{\rm C}$ factor and, in addition, illustrates in the diagrams the degree of influence on contamination of the different influencing factors. When $e_{\rm C}$ is determined, it is used for calculating the life modification factor according to 9.3.3.4.

The contamination factors can be determined by means of either the diagrams or the equations for the following lubrication methods:

- circulating oil lubrication with the oil filtered on-line before being supplied to the bearings,
- oil bath lubrication or circulating oil lubrication with off-line filters,
- grease lubrication.

For estimating the influence of contamination on the $e_{\rm C}$ factor when oil mist lubrication is used, see ISO/TR 1281^[2].

A.2 Symbols

For the purposes of this annex, the symbols in Clause 4 and the following apply.

- contamination particle size, in μ m(c) with ISO 11171^[5] calibration
- $\beta_{x(c)}$ filtration ratio at contamination particle size x (see symbol x above)

The designation (c) signifies that the particle counters — of particles of size $x \mu m$ — shall be APC (automatic optical single-particle counter) calibrated in accordance with ISO 11171^[5].

A.3 Conditions for the selection of diagrams and equations for different lubricating methods

A.3.1 Circulating oil with on-line filters

The filtration ratio $\beta_{x(c)}$, with particle size x in μ m(c) according to ISO 16889^[6], is the most influencing factor when selecting diagrams and equations. The applicable contamination level for a range of cleanliness codes according to ISO 4406^[7] is also indicated in these figures. The contamination level corresponds mainly to the condition of the oil before it passes the on-line filter.

NOTE Research involving the accuracy of measuring oil cleanliness by means of sampled oils has resulted in the conclusion that it is extremely difficult to determine oil cleanliness with any degree of accuracy. Even by taking every precaution, it is difficult not to pollute the sampled oil, and in addition there is a risk of including precipitated oil additives in the particle calculation. The risk of obtaining an incorrect measuring result due to external pollution is largest when very clean oils are analysed.

The cleanliness of circulation oil in applications with on-line filter normally increases when the oil has passed through the filter for a certain period of time. Therefore, the general contamination level of the oil before it passes the on-line filter may provide the best representation of the actual oil cleanliness in circulating oil systems. The difficulty associated with measuring oil cleanliness accurately is the reason for using the filtration ratio $\beta_{x(c)}$ with particle size x as the main influencing factor when selecting the proper e_C diagram or equation for on-line circulating oil systems.

A.3.2 Oil bath lubrication

For oil bath and circulating oil systems with off-line filters only, the selection of diagrams and equations is determined by the required contamination level, given as a range of cleanliness codes according to ISO 4406.

A.3.3 Grease lubrication

For grease lubrication, the recommended selection of diagrams and equations for different cleanliness levels are indicated in Table A.1, and the selection shall be based on this table only.

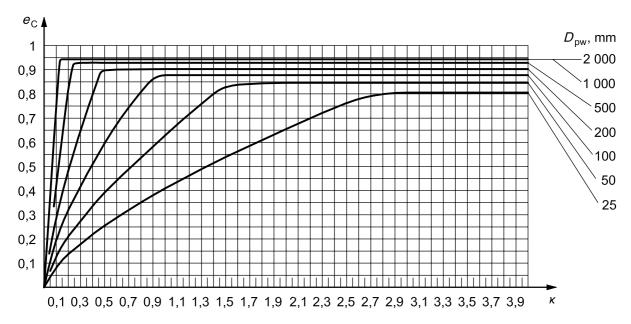
A.3.4 Bearing mounting and oil supply

To obtain predicted bearing lives, it is important to have the bearing running under expected operating conditions already from the start and after supplying new oil to the lubrication system.

Careful flushing of the bearing application after mounting is therefore important, with especially strong demands when the bearings are operating under expected cleanest conditions. It is also important that new oils are filtered before being supplied to the oil system. The filter shall then be at least as good as, but preferably more efficient than, filters used for the oil system.

A.4 Contamination factor, $e_{\rm C}$, for circulating oil lubrication with on-line filters

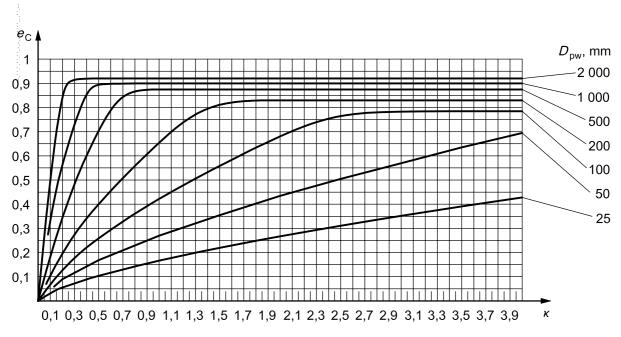
For circulating oil systems with on-line filters, before the oil is supplied to the bearings, the contamination factor, $e_{\rm C}$, can be determined by means of the diagrams or the equations in Figures A.1 to A.4. Primarily the filtration ratio, $\beta_{x({\rm C})}$, determines the selection of diagram or equation, and for a selected $x({\rm C})$, the $\beta_{x({\rm C})}$ value shall then be as high as or higher than the value indicated for each diagram. The oil system shall also have cleanliness within the range indicated by the cleanliness code according to ISO 4406.



Equation:
$$e_{\rm C}=a\left(1-\frac{0,5663}{D_{\rm pw}^{-1/3}}\right)$$
, where $a=0,0864~\kappa^{-0,68}~D_{\rm pw}^{-0,55}$, with the restriction $a\leqslant 1$

Range of ISO 4406 codes: —/13/10, —/12/10, —/13/11, —/14/11

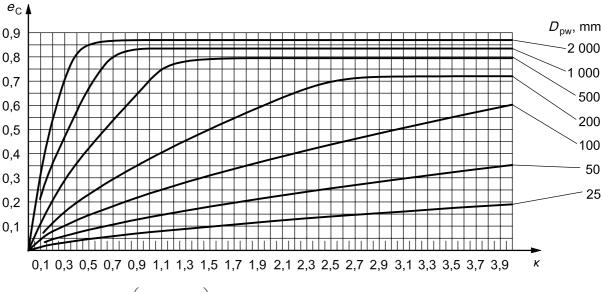
Figure A.1 — $e_{\rm C}$ factor for circulating oil lubrication with on-line filters – $\beta_{\rm 6(c)}$ = 200, ISO 4406 code —/13/10



Equation:
$$e_{\rm C}=a\left(1-\frac{0.9987}{D_{\rm pw}^{-1/3}}\right)$$
, where $a=0.0432~\kappa^{-0.68}~D_{\rm pw}^{-0.55}$, with the restriction $a\leqslant 1$

Range of ISO 4406 codes: —/15/12, —/16/12, —/15/13, —/16/13

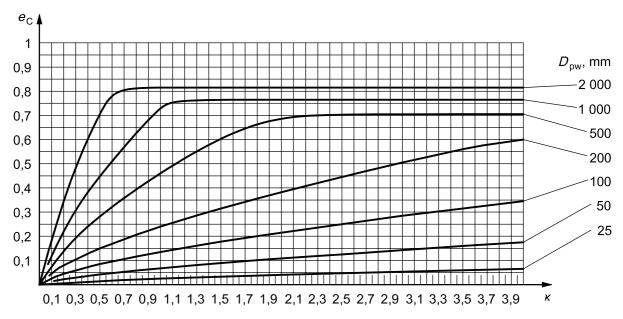
Figure A.2 — $e_{\rm C}$ factor for circulating oil lubrication with on-line filters – $\beta_{12(c)}$ = 200, ISO 4406 code —/15/12



Equation:
$$e_{\rm C} = a \left(1 - \frac{1,6329}{D_{\rm pw}^{-1/3}} \right)$$
, where $a = 0,0288 \, \kappa^{-0.68} \, D_{\rm pw}^{-0.55}$, with the restriction $a \le 1$

Range of ISO 4406 codes: —/17/14, —/18/14, —/18/15, —/19/15

Figure A.3 — $e_{\rm C}$ factor for circulating oil lubrication with on-line filters – $\beta_{25(c)} \ge 75$, ISO 4406 code —/17/14



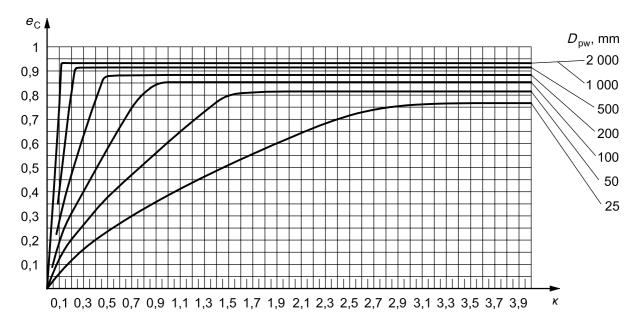
Equation:
$$e_{\rm C}=a\left(1-\frac{2{,}3362}{D_{\rm pw}^{-1/3}}\right)$$
, where $a=0{,}0216~\kappa^{-0.68}~D_{\rm pw}^{-0.55}$, with the restriction $a\leqslant 1$

Range of ISO 4406 codes: —/19/16, —/20/17, —/21/18, —/22/18

Figure A.4 — $e_{\rm C}$ factor for circulating oil lubrication with on-line filters – $\beta_{40(c)} \ge 75$, ISO 4406 code —/19/16

A.5 Contamination factor, $e_{\rm C}$, for oil lubrication without filtration or with off-line filters

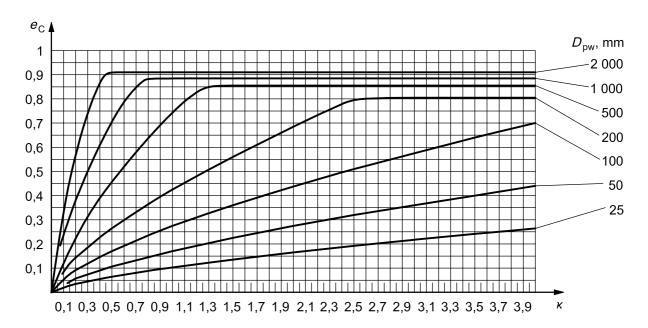
For oil lubrication without filtration or with off-line filters, the contamination factor, $e_{\rm C}$, can be determined by means of the diagrams or the equations in Figures A.5 to A.9. The range of cleanliness codes, according to ISO 4406, indicated for each figure is to be used for the selection of a suitable diagram or equation.



Equation:
$$e_{\rm C} = a \left(1 - \frac{0,6796}{D_{\rm pw}^{-1/3}} \right)$$
, where $a = 0,0864 \, \kappa^{0,68} \, D_{\rm pw}^{-0,55}$, with the restriction $a \leqslant 1$

Range of ISO 4406 codes: —/13/10, —/12/10, —/11/9, —/12/9

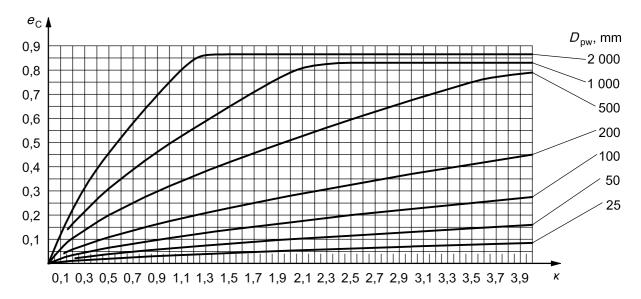
Figure A.5 — $e_{\rm C}$ factor for oil lubrication without filtration or with off-line filters – ISO 4406 code —/13/10



Equation:
$$e_{\rm C}=a\left(1-\frac{1,141}{D_{\rm pw}^{-1/3}}\right)$$
, where $a=0,0288~\kappa^{-0,68}~D_{\rm pw}^{-0,55}$, with the restriction $a\leqslant 1$

Range of ISO 4406 codes: —/15/12, —/14/12, —/16/13, —/16/13

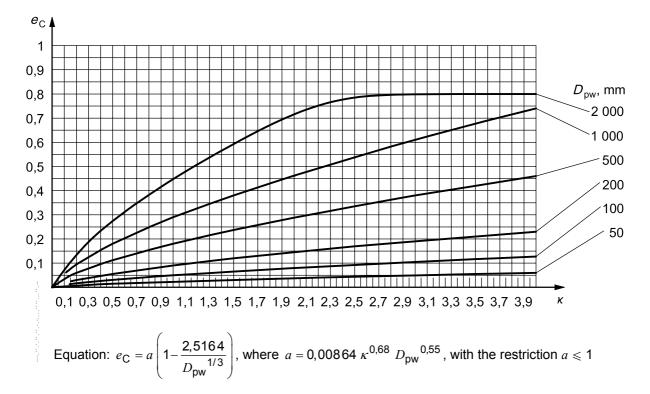
Figure A.6 — $e_{\rm C}$ factor for oil lubrication without filtration or with off-line filters – ISO 4406 code —/15/12



Equation:
$$e_{\rm C} = a \left(1 - \frac{1,67}{D_{\rm pw}^{-1/3}} \right)$$
, where $a = 0,0133 \, \kappa^{0,68} \, D_{\rm pw}^{-0,55}$, with the restriction $a \leq 1$

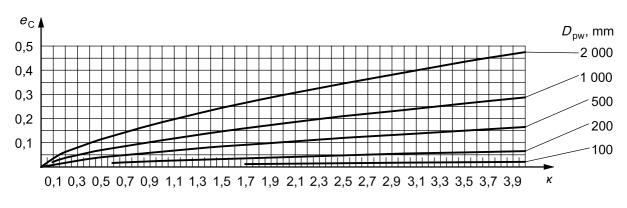
Range of ISO 4406 codes: —/17/14, —/18/14, —/18/15, —/19/15

Figure A.7 — $e_{\rm C}$ factor for oil lubrication without filtration or with off-line filters – ISO 4406 code —/17/14



Range of ISO 4406 codes: -/19/16, -/18/16, -/20/17, -/21/17

Figure A.8 — $e_{\rm C}$ factor for oil lubrication without filtration or with off-line filters – ISO 4406 code —/19/16



Equation:
$$e_{\rm C}=a\left(1-\frac{3,8974}{D_{\rm pw}^{-1/3}}\right)$$
, where $a=0,00411\,\kappa^{0,68}\,D_{\rm pw}^{-0,55}$, with the restriction $a\leqslant 1$

Range of ISO 4406 codes: —/21/18, —/21/19, —/22/19, —/23/19

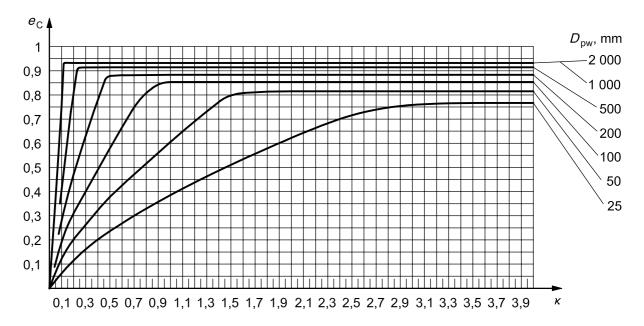
Figure A.9 — $e_{\rm C}$ factor for oil lubrication without filtration or with off-line filters – ISO 4406 code —/21/18

A.6 Contamination factor, $e_{\rm C}$, for grease lubrication

For grease lubrication, the contamination factor, $e_{\rm C}$, can be determined by means of the diagrams or the equations in Figures A.10 to A.14. Table A.1 is to be used for the selection of a suitable diagram or equation. Select the operating condition row in the table that most fully represents the existing conditions.

Table A.1 — Selection of diagrams and equations for grease lubrication

Operating conditions	Level of contamination
Very clean assembly with careful flushing; very good sealing in relation to operating conditions; regreasing carried out continuously or at short intervals	High cleanliness Figure A.10
Sealed bearings, greased for life with effective sealing capacity in relation to operating conditions	
Clean assembly with flushing; good sealing in relation to operating conditions; regreasing according to manufacturer's specification	Normal cleanliness Figure A.11
Sealed bearings, greased for life with proper sealing capacity in relation to operating conditions, e.g. shielded bearings	
Clean assembly; moderate sealing capacity in relation to operating conditions; regreasing according to manufacturer's specifications	Slight to typical contamination Figure A.12
Assembly in workshop; bearing and application not adequately washed after mounting; poor sealing capacity in relation to operating conditions; regreasing intervals longer than recommended by manufacturer	Severe contamination Figure A.13
Assembly in contaminated environment; inadequate sealing; long regreasing intervals	Very severe contamination Figure A.14



Equation:
$$e_{\rm C}=a\left(1-\frac{0,6796}{D_{\rm pw}^{-1/3}}\right)$$
, where $a=0,0864~\kappa^{-0,68}~D_{\rm pw}^{-0,55}$, with the restriction $a\leqslant 1$

Figure A.10 — e_{C} factor for grease lubrication — High cleanliness

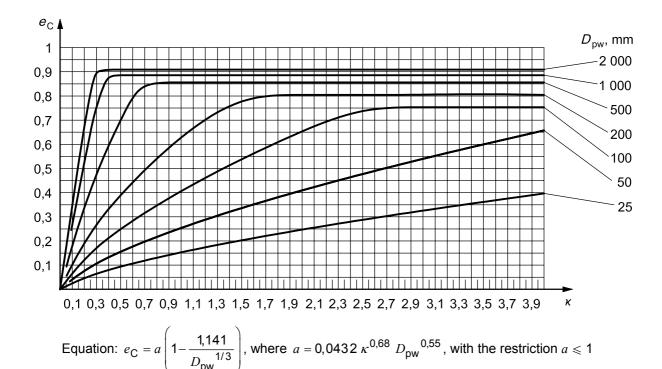
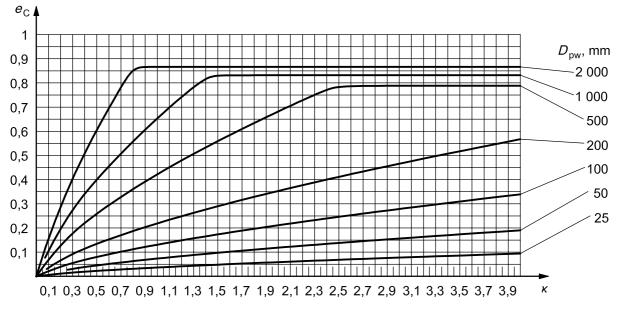
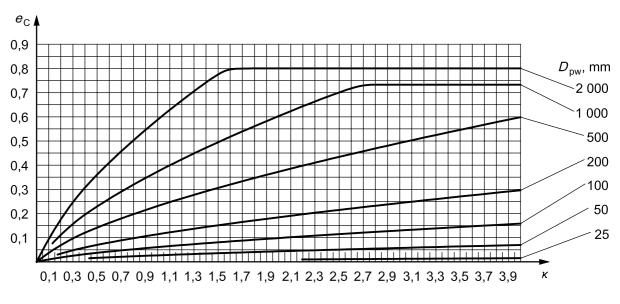


Figure A.11 — $e_{\rm C}$ factor for grease lubrication — Normal cleanliness



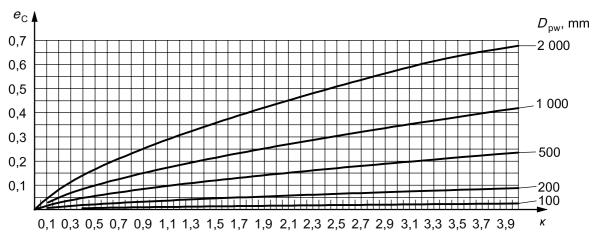
Equations:

Figure A.12 — $e_{\mathbb{C}}$ factor for grease lubrication — Slight to typical contamination



Equation:
$$e_{\rm C} = a \left(1 - \frac{2,662}{D_{\rm pw}^{-1/3}} \right)$$
, where $a = 0,0115 \, \kappa^{0,68} \, D_{\rm pw}^{-0,55}$, with the restriction $a \leqslant 1$

Figure A.13 — $e_{\rm C}$ factor for grease lubrication — Severe contamination



Equation:
$$e_{\rm C} = a \left(1 - \frac{4,06}{D_{\rm pw}^{-1/3}} \right)$$
, where $a = 0,00617 \, \kappa^{0,68} \, D_{\rm pw}^{-0,55}$, with the restriction $a \leq 1$

Figure A.14 — $e_{\rm C}$ factor for grease lubrication — Very severe contamination

Annex B

(informative)

Calculation of the fatigue load limit

B.1 General

This annex contains recommendations for the calculation of the fatigue load limit, $C_{\rm u}$, considering bearing type, size and bearing internal geometry, the profile of rolling elements and raceways and the fatigue limit of the raceway material.

For the application of this procedure, the directions and limitations given in this International Standard apply.

The fatigue load limit, $C_{\rm u}$, is not to be used as exclusive criteria for bearing selection. Rolling bearings will not necessarily have an infinite life at bearing loads below the fatigue limit. In practical applications of rolling bearings, boundary or mixed lubrication and lubricant contamination can lead to increased stresses in the raceway material, so that even in case of a bearing load below the fatigue load limit, the fatigue limit of the raceway material can be exceeded locally. Such effects of lubrication and lubricant contamination are taken into account by the life rating methods in 9.3 and Annex A.

B.2 Symbols

For the purposes of this annex, the symbols in Clause 4 and the following apply.

- E modulus of elasticity, in newtons per square millimetre
- $E(\chi)$ complete elliptic integral of the second kind
- e subscript for outer ring or housing washer
- $F(\rho)$ relative curvature difference
- i subscript for inner ring or shaft washer
- $K(\chi)$ complete elliptic integral of the first kind
- $Q_{\rm II}$ fatigue load limit of a single contact, in newtons
- re cross-sectional raceway groove radius of outer ring, in millimetres
- r_i cross-sectional raceway groove radius of inner ring, in millimetres
- χ ratio of semi major to semi minor axis of the contact ellipse
- $\gamma \qquad \qquad \text{auxiliary parameter, } \gamma = \frac{D_{\text{W}} \cos \alpha}{D_{\text{DW}}}$
- φ angular position of rolling element, in degrees
- ν_{E} Poisson's ratio

 ρ curvature of contact surface, in millimetres to the power of minus one

 $\sum
ho$ curvature sum, in millimetres to the power of minus one

 σ_{Hu} Hertzian contact stress at which the fatigue limit of the raceway material is reached, in newtons per square millimetre

B.3 Fatigue load limit, C_{11}

B.3.1 General

The life modification factor, $a_{\rm ISO}$, can be expressed as a function of the ratio $C_{\rm u}/P$, i.e. the fatigue load limit divided by the dynamic equivalent load, as explained in 9.3.2.

An advanced method for calculating the fatigue load limit, $C_{\rm u}$, of a bearing is shown in B.3.2. A contact stress of 1 500 MPa⁴) between rolling elements and raceways has been applied. This contact stress is recommended for rolling bearings of commonly used high quality material and good manufacturing quality.

For a rough estimation of C_{II} , a simplified method is presented in B.3.3.

B.3.2 Advanced method for calculating the fatigue load limit, $C_{\rm II}$

B.3.2.1 Fatigue load limit of a single contact

B.3.2.1.1 General

The fatigue load limit of a single contact is the load, at which the stress in the raceway material just reaches the fatigue limit of this material. For point contact, this load can be calculated analytically, while profiled line contact requires a more complex numerical analysis.

B.3.2.1.2 Ball bearings

For the calculation of the fatigue load limit, the actual curvature radii of ball and raceways shall be used.

The fatigue load limit at a single inner ring [shaft washer] raceway contact and a single outer ring [housing washer] raceway contact is calculated as

$$Q_{\text{u i, e}} = \sigma_{\text{Hu}}^{3} \times \frac{32\pi \chi_{\text{i, e}}}{3} \left(\frac{1 - v_{\text{E}}^{2}}{E} \times \frac{E(\chi_{\text{i, e}})}{\sum \rho_{\text{i, e}}} \right)^{2}$$
(B.1)

The ratio of semi major to semi minor axis of the contact ellipse can be derived from Equation (B.2).

$$1 - \frac{2}{\chi^2 - 1} \left(\frac{K(\chi)}{E(\chi)} - 1 \right) - F(\rho) = 0$$
 (B.2)

⁴⁾ $1 \text{ MPa} = 1 \text{ N/mm}^2$.

The complete elliptic integral of the first kind in Equation (B.2) is

$$K(\chi) = \int_{0}^{\frac{\pi}{2}} \left[1 - \left(1 - \frac{1}{\chi^2} \right) (\sin \varphi)^2 \right]^{-\frac{1}{2}} d\varphi$$
 (B.3)

and the complete elliptic integral of the second kind is

$$E(\chi) = \int_{0}^{\frac{\pi}{2}} \left[1 - \left(1 - \frac{1}{\chi^2} \right) (\sin \varphi)^2 \right]^{\frac{1}{2}} d\varphi \tag{B.4}$$

The curvature sum of the inner ring [shaft washer] raceway contacts in Equation (B.1) is

$$\sum \rho_{\mathbf{i}} = \frac{2}{D_{\mathbf{W}}} \left(2 + \frac{\gamma}{1 - \gamma} - \frac{D_{\mathbf{W}}}{2 r_{\mathbf{i}}} \right) \tag{B.5}$$

and the curvature sum of the outer ring [housing washer] contacts is

$$\sum \rho_{\mathsf{e}} = \frac{2}{D_{\mathsf{w}}} \left(2 - \frac{\gamma}{1 + \gamma} - \frac{D_{\mathsf{w}}}{2r_{\mathsf{e}}} \right) \tag{B.6}$$

The relative curvature difference of the inner ring [shaft washer] raceway contacts is

$$F_{i}(\rho) = \frac{\frac{\gamma}{1 - \gamma} + \frac{D_{w}}{2r_{i}}}{2 + \frac{\gamma}{1 - \gamma} - \frac{D_{w}}{2r_{i}}}$$
(B.7)

and the relative curvature difference of the outer ring [housing washer] raceway contacts is

$$F_{e}(\rho) = \frac{\frac{-\gamma}{1+\gamma} + \frac{D_{w}}{2r_{e}}}{2 - \frac{\gamma}{1+\gamma} - \frac{D_{w}}{2r_{e}}}$$
(B.8)

When the fatigue load limits of the most heavily loaded contact on inner ring [shaft washer] raceway, $Q_{\rm ui}$, and outer ring [housing washer] raceway, $Q_{\rm ue}$, are calculated, the actual contact geometry is considered, i.e. the curvature radii of ball and raceway.

When the fatigue load limit, $C_{\rm u}$, is calculated, the smallest value of the two calculated values, $Q_{\rm ui}$ and $Q_{\rm ue}$, is applied, i.e.

$$Q_{\mathsf{u}}^{\dagger} = \min\left(Q_{\mathsf{u}\mathsf{i}}, Q_{\mathsf{u}\mathsf{e}}\right) \tag{B.9}$$

For self-aligning ball bearings, a 60 % higher fatigue load limit than the corresponding value for radial ball bearings is permitted for the outer ring raceway contact. In analogy with the static load ratings in ISO 76, a higher contact stress can be accepted in the outer ring raceway contact.

B.3.2.1.3 Roller bearings

When the fatigue load limits of the most heavily loaded contact on inner ring [shaft washer] raceway, $Q_{\rm ui}$, and on outer ring [housing washer] raceway, $Q_{\rm ue}$, are calculated, the actual contact geometry is considered, i.e. the curvature radii and profiles of rolling element and raceway.

The calculation of contact stress in profiled line contact requires a complex numerical analysis. Suitable calculation methods are described in References [8], [9] and [10]. The Hertzian equations for line contact of cylindrical bodies in Reference [11] are not adequate.

B.3.2.2 Fatigue load limit of a complete bearing

B.3.2.2.1 General

The fatigue load limit, $C_{\rm u}$, of a complete bearing is determined by inserting the minimum fatigue load limit of the highest loaded contact $Q_{\rm u}$ [see Equation (B.9)] in Equations (B.10) to (B.17) below.

B.3.2.2.2 Radial ball bearings

$$C_{\rm u} = 0,228 \, 8 \, Z \, Q_{\rm u} \, i \cos \alpha$$
 for $D_{\rm pw} \leqslant 100 \, {\rm mm}$ (B.10)

$$C_{\rm u} = 0,2288ZQ_{\rm u}i\cos\alpha \left(\frac{100}{D_{\rm pw}}\right)^{0.5}$$
 for $D_{\rm pw} > 100~{\rm mm}$ (B.11)

B.3.2.2.3 Thrust ball bearings

$$C_{\rm u} = Z Q_{\rm u} \sin \alpha$$
 for $D_{\rm DW} \leqslant 100 \, {\rm mm}$ (B.12)

$$C_{\rm u} = Z Q_{\rm u} \sin \alpha \left(\frac{100}{D_{\rm pw}}\right)^{0.5}$$
 for $D_{\rm pw} > 100 \text{ mm}$ (B.13)

B.3.2.2.4 Radial roller bearings

$$C_{\rm u} = 0.245 \, 3 \, Z \, Q_{\rm u} \, i \cos \alpha$$
 for $D_{\rm pw} \leqslant 100 \, {\rm mm}$ (B.14)

$$C_{\rm u} = 0.2453 Z Q_{\rm u} i \cos \alpha \left(\frac{100}{D_{\rm pw}}\right)^{0.3}$$
 for $D_{\rm pw} > 100 \text{ mm}$ (B.15)

B.3.2.2.5 Thrust roller bearings

$$C_{\text{U}} = Z Q_{\text{U}} \sin \alpha$$
 for $D_{\text{DW}} \leqslant 100 \text{ mm}$ (B.16)

$$C_{\rm u} = Z Q_{\rm u} \sin \alpha \left(\frac{100}{D_{\rm pw}}\right)^{0,3}$$
 for $D_{\rm pw} > 100 \text{ mm}$ (B.17)

B.3.3 Simplified method for calculating the fatigue load limit, $C_{\rm u}$

B.3.3.1 General

For a simplified estimation of the fatigue load limit, $C_{\rm u}$, for ball bearings and roller bearings, Equations (B.18) to (B.21) can be used.

The results of this simplified estimation can differ significantly from the results of the advanced method given in B.3.2. The results of the advanced method are preferred.

B.3.3.2 Ball bearings

$$C_{\rm u} = \frac{C_0}{22}$$
 for bearings with $D_{\rm pw} \leqslant 100$ mm (B.18)

$$C_{\rm u} = \frac{C_0}{22} \left(\frac{100}{D_{\rm pw}}\right)^{0.5}$$
 for bearings with $D_{\rm pw} > 100$ mm (B.19)

B.3.3.3 Roller bearings

$$C_{\rm u} = \frac{C_0}{8.2}$$
 for bearings with $D_{\rm pw} \leqslant 100$ mm (B.20)

$$C_{\rm u} = \frac{C_0}{8.2} \left(\frac{100}{D_{\rm pw}}\right)^{0.3}$$
 for bearings with $D_{\rm pw} > 100$ mm (B.21)

The ratio $C_0/C_u = 8.2$ accounts in part for roller profile. NOTE

Annex C (informative)

Discontinuities in the calculation of basic dynamic load ratings

C.1 General

The factors used for calculation of basic dynamic load ratings $C_{\rm r}$ and $C_{\rm a}$, according to this International Standard are slightly different for radial and thrust angular contact ball bearings. The methods for taking into account the influence of axial loads on bearing life are also different.

Therefore there is a discontinuity in the calculated lives when a bearing with the contact angle $\alpha = 45^{\circ}$ is first regarded as a radial bearing and then as a thrust bearing. In both cases, the bearing is subject to the same external axial load F_a only.

This annex explains why the load rating factors for calculation of the basic dynamic load ratings $C_{\rm r}$ and $C_{\rm a}$ are different, and shows how these load ratings can be recalculated in order to bring about correct comparisons under the same conditions.

C.2 Symbols

For the purposes of this annex, the symbols in Clause 4 and the following apply.

- $C_{\rm aa}$ adjusted basic dynamic axial load rating for a thrust bearing ($\alpha > 45^{\circ}$), in newtons
- C_{ar} adjusted basic dynamic axial load rating for a radial bearing ($\alpha \leq 45^{\circ}$), in newtons
- $r_{
 m e}$ cross-sectional raceway groove radius of outer ring, in millimetres
- r_i cross-sectional raceway groove radius of inner ring, in millimetres
- λ contact stress factor

C.3 Different factors for calculating load rating and equivalent load for radial and thrust angular contact ball bearings

When a life comparison is made between a radial and a thrust bearing, both bearings are assumed to be subject to the same external axial load F_a only.

a) For angular contact thrust ball bearings

$$L_{10} = \left(\frac{C_{\mathsf{a}}}{P_{\mathsf{a}}}\right)^3 = \left(\frac{C_{\mathsf{a}}}{F_{\mathsf{a}}}\right)^3$$

- Included in the calculation of C_a are
 - the conformities between balls and raceways $r_i/D_w \le 0.54$ and $r_e/D_w \le 0.54$,
 - a contact stress factor $\lambda = 0.9$,

— the *Y* factor ($C_a = C_f/Y$), where

$$Y = \frac{0.4 \cot \alpha}{1 - 0.333 \sin \alpha} \tag{C.1}$$

b) For angular contact radial ball bearings

with
$$C_{\mathsf{a}} = \frac{C_{\mathsf{r}}}{Y}$$

$$L_{10} = \left(\frac{C_{\mathsf{r}}}{P_{\mathsf{r}}}\right)^3 = \left(\frac{C_{\mathsf{r}}}{YF_{\mathsf{a}}}\right)^3 = \left(\frac{C_{\mathsf{a}}}{F_{\mathsf{a}}}\right)^3 \tag{C.2}$$

- Included in the calculation of C_r are
 - the conformities between balls and raceways $r_i/D_w \le 0.52$ and $r_e/D_w \le 0.53$,
 - a contact stress factor $\lambda = 0.95$.

The Y factor is calculated according to Equation (C.1) if all balls are loaded, as is mostly the case for thrust bearings. The expression $1-0.333 \sin \alpha$ in Equation (C.1) takes into consideration the negative influence of the fact that all balls are loaded and is included in the $f_{\rm c}$ values for angular contact thrust ball bearings in Table 4.

Radial bearings are mainly radially loaded and many balls are unloaded or lightly loaded. The negative influence of the expression $1 - 0.333 \sin \alpha$ was therefore reduced when the Y factors were calculated for angular contact radial ball bearings in Table 3.

C.4 Comparison of adjusted basic dynamic axial load ratings $C_{\rm ar}$ and $C_{\rm aa}$ for radial and thrust angular contact ball bearings

C.4.1 General

For certain applications, angular contact ball bearings with contact angles $\alpha \le 45^{\circ}$ and $\alpha > 45^{\circ}$ are manufactured with the same conformity between balls and raceways, and sometimes there is a need to calculate and also to compare their true axial load ratings.

The basic dynamic load ratings C_r and C_a can be calculated using this International Standard or taken from a bearing catalogue if they are available there.

However, as described in C.3, $C_{\rm r}$ and $C_{\rm a}$ are calculated with different values of conformity, λ factor and Y factor for radial and thrust bearings. If a correct calculation and comparison is to be made, $C_{\rm r}$ and $C_{\rm a}$ have to be recalculated to adjusted basic dynamic axial load ratings $C_{\rm ar}$ and $C_{\rm aa}$, based upon the same values of conformity, λ factor and Y factor.

The recalculation can be performed using Equations (C.3), (C.4), (C.7) and (C.8) for two different conformities — radial bearing and thrust bearing conformities — as defined in 5.1 and 6.1.1.

Comparison of load ratings is mainly of interest for bearings intended to operate in applications where axial loads are predominant, and therefore comparison of basic dynamic axial load ratings is dealt with in this annex.

The contact angle α is assumed to be constant, independent of the axial load, which means that the accuracy is reduced for bearings with small contact angles, subjected to heavy loads.

C.4.2 Angular contact ball bearings with radial bearing conformities

 $(r_{\rm j}/D_{\rm W} \leqslant 0.52 \ {\rm and} \ r_{\rm e}/D_{\rm W} \leqslant 0.53)$

$$C_{ar} = 2,37 \tan \alpha (1 - 0,333 \sin \alpha) C_r$$
 (C.3)

$$C_{aa} = 1,24 C_a$$
 (C.4)

$$L_{10} = \left(\frac{C_{\text{ar}}}{F_{\text{a}}}\right)^3 \tag{C.5}$$

$$L_{10} = \left(\frac{C_{aa}}{F_a}\right)^3 \tag{C.6}$$

C.4.3 Angular contact ball bearings with thrust bearing conformities

 $(r_i/D_w \leqslant 0.54 \text{ and } r_e/D_w \leqslant 0.54)$

$$C_{ar} = 1.91 \tan \alpha (1 - 0.333 \sin \alpha) C_r$$
 (C.7)

$$C_{\mathsf{a}\mathsf{a}} = C_{\mathsf{a}} \tag{C.8}$$

C.5 Examples

C.5.1 Angular contact ball bearing with $\alpha = 45^{\circ}$

Compare the adjusted basic dynamic axial load ratings $C_{\rm ar}$ and $C_{\rm aa}$ of an angular contact ball bearing with α = 45°, when it is regarded as a radial bearing and as a thrust bearing. For the selected bearing, $(D_{\rm W}\cos\alpha)/D_{\rm DW}=0.16$ and i=1. The bearing has radial bearing conformities.

As a radial bearing

 $C_{\rm r}$ is calculated according to Equation (1), i.e. $C_{\rm r} = K f_{\rm c}$, where K is a factor, which includes all parameters that are the same for this radial and thrust bearing. According to Table 2, $f_{\rm c} = 59,6$.

Equation (C.3) gives

$$C_{ar} = 2,37 \times \tan 45^{\circ} \times (1 - 0,333 \sin 45^{\circ}) \times K \times 59,6 = 108 K$$

As a thrust bearing

 C_a is calculated according to Equation (6), i.e. $C_a = K f_c \tan \alpha$. The factor $f_c = 85,1$ from Table 4.

Equation (C.4) gives

$$C_{aa} = 1,24 \times K \times 85,1 \times \tan 45^{\circ} = 106 K$$

These calculations show that the basic dynamic load ratings $C_{ar} \approx C_{aa}$, which confirms that there is no discontinuity.

C.5.2 Angular contact ball bearing with $\alpha = 40^{\circ}$

Calculate the adjusted basic dynamic axial load rating $C_{\rm ar}$ of a single-row angular contact ball bearing with the contact angle $\alpha = 40^{\circ}$. The bearing has thrust bearing conformities. $D_{\rm W}/D_{\rm pw} = 0.091$, ball diameter $D_{\rm w}$ = 7,5 mm and the number of balls Z = 27.

According to Table 2, for $(D_{\rm W}\cos40^\circ)/D_{\rm DW}=0.091\times\cos40^\circ=0.07$, and then $f_{\rm C}=51.1$.

Equation (1) gives

$$C_r = 1.3 f_c (\cos \alpha)^{0.7} Z^{2/3} D_w^{1.8} = 1.3 \times 51.1 \times (\cos 40^\circ)^{0.7} \times 27^{2/3} \times 7.5^{1.8} = 18651.$$

NOTE This load rating is based on radial bearing conformities.

According to Equation (C.7),

$$C_{ar} = 1,91 \times \tan 40^{\circ} \times (1 - 0,333 \times \sin 40^{\circ}) \times 18 651 = 23 493$$

$$C_{ar} = 23\,500\,N$$

C.5.3 Angular contact ball bearing with $\alpha = 60^{\circ}$

Calculate the adjusted basic dynamic axial load rating C_{aa} of a single-row angular contact ball bearing with the contact angle $\alpha = 60^{\circ}$. The bearing has thrust bearing conformities. $D_{\rm W}/D_{\rm DW} = 0{,}091$, ball diameter $D_{\rm w}$ = 7,5 mm and the number of balls Z = 27.

According to Table 4, for $(D_{\rm W}\cos 60^{\circ})/D_{\rm DW} = 0.091 \times \cos 60^{\circ} = 0.046$, and then $f_{\rm C} = 61.12$.

Equation (6) gives

$$C_a = 1.3 f_c (\cos \alpha)^{0.7} (\tan \alpha) Z^{2/3} D_w^{1.8} = 1.3 \times 61.12 \times (\cos 60^\circ)^{0.7} \times \tan 60^\circ \times 27^{2/3} \times 7.5^{1.8} = 28.663.$$

NOTE This load rating is based on thrust bearing conformities.

According to Equation (C.8),

$$C_{aa} = C_a = 28700 \text{ N}.$$

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⁵⁾ In preparation.

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